Efficient Combinatorial Allocations: Individual Rationality versus Stability[1]

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Abstract

We investigate combinatorial allocations with opt-out types and clarify the possibility of achieving efficiency under incomplete information. We introduce two distinct collective decision procedures. The first procedure assumes that the central planner designs a mechanism and players have the option to exit. The mechanism requires interim individual rationality. The second procedure assumes that players design a mechanism by committing themselves to participate. The mechanism requires marginal stability against blocking behavior by the largest proper coalitions. We show that the central planner can earn non-negative revenue in the first procedure, if and only if he cannot do so in the second.

Keywords: Efficient Combinatorial Allocations, Bayesian Incentive Compatibility, Opt-Out Types, Interim Individual Rationality, Marginal Stability.

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1. Introduction

This paper investigates the allocation problem with incomplete information, with the assumptions of quasi-linearity, private values, and independent distributions, where each player has a type that is unknown to the other players and the central planner. Our framework is general enough to include combinatorial auctions[3], multilateral trading[4], and incentive auctions[5] in which each participant (player) brings heterogeneous commodities as his (or her) initial endowment to sell to the other participants and purchases other commodities at the same time. Our framework includes allocation problems in which both the central planner and the participants bring their respective commodities and sell them altogether. For example, broadcast television companies sell spectrum licenses to mobile phone companies, and the government sells substitute licenses to these broadcast television companies simultaneously. The purpose of this paper is to clarify the possibility of achieving efficient allocations without the central planner having a deficit in revenue.

We introduce and compare the following two distinct collective decision procedures, both of which require a mechanism to satisfy efficiency and *Bayesian incentive compatibility* (BIC). The first collective decision procedure assumes that the central planner has the initiative in designing the mechanism at the ex-ante stage. At the interim stage, after observing his (or her) type, each player decides whether to participate in or exit from the procedure. Whenever he (or she) decides to exit from the procedure, he can consume his initial endowment by himself. The central planner needs to incentivize all players to participate in the procedure irrespective of their types, that is, the mechanism requires *interim individual rationality* (IIR) in the first collective decision procedure.

The second collective decision procedure assumes that the central planner transfers the initiative in designing the mechanism to the players at the ex-ante stage. The players commit themselves not to exit from the procedure irrespective of their types; the

3 See Rassenti, Bulfin, and Smith (1982), Kelso and Crawford (1982), and Ausubel and Milgrom (2002). See also Cramton et al. (2006).
4 See Myerson and Satterthwaite (1983).
5 See Milgrom (2007), Cramton (2011), Hazlett et al. (2012), and Milgrom et al. (2012).

mechanism does not require IIR. However, in the second collective decision procedure, we require the mechanism to satisfy the following stability notion against coalitional behavior for occupying the central planner's endowment, namely *marginal stability* (MS). At the ex-ante stage, any largest proper coalition has the option to exclude the player who does not belong to this coalition from the allocation problem. MS requires a mechanism to incentivize any largest proper coalition not to exercise this option.

Following Makowski and Ostroy (1989) and Segal and Whinston (2010), we assume that there is an *opt-out type* in the set of possible types for each player, with which, the consumption of his initial endowment is valuable to the point that the efficient allocation assigns it to him irrespective of other players' types. With the presence of such opt-out types, we show that there exists an efficient, BIC, and marginally stable mechanism that brings the central planner a non-negative revenue, if and only if, any efficient, BIC, and interim individual rational mechanism never brings the central planner a non-negative revenue. Hence, it follows that the first collective decision procedure functions to make the achievement of efficiency consistent with the central planner's non-negative revenue, if and only if, the second collective decision procedure does not do so.

In the context of combinatorial auctions, when the central planner brings the entire set of commodities to be sold, each player's outside opportunity could be low enough to be consistent with IIR, because he has nothing to consume by exiting. However, in the second collective decision procedure, the players are willing to build a coalition to exclude its non-member from accessing the central planner's endowment. The requirement of MS is restrictive in this context.

In the context of multilateral trading, where the players bring the entire set of commodities to be sold, each player's outside opportunity could be too high to be consistent with IIR. However, in the second collective decision procedure, the players hesitate to build any coalition, because they dislike losing the chance to trade with its non-member. The requirement of MS is not restrictive in this context.

This paper is in line with Segal and Whinston (2010), which assumed the presence of opt-out types, and then showed a necessary condition under which the first collective decision procedure functions to make the achievement of efficiency consistent with the central planner's non-negative revenue. The present paper shows a not only necessary but also sufficient condition under which the first collective decision procedure functions to make the achievement of efficiency consistent with the central planner's non-negative revenue, by introducing the second collective decision procedure and comparing the two. The present paper also allows the central planner to bring commodities to sell alongside the participants' endowments.

The organization of this paper is as follows. Section 2 describes a basic model of allocation problems. Section 3 explains combinatorial allocations, introduces opt-out types, and demonstrates the characterization result for the first collective decision procedure. Section 4 defines the second collective decision procedure, defines MS, and shows the main result of this paper. Section 5 discusses the impact of replacing MS with the standard notion of stability. Section 6 concludes.

2. The Basic Model

This paper investigates an allocation problem with incomplete information, where A denotes the set of all allocations, $N \equiv \{1,2,...,n\}$ denotes the set of all players, with $a \in A$ and $n \geq 2$. Each player $i \in N$ has a *type* $\omega_i \in \Omega_i$ that is unknown to the other players and the central planner, where Ω_i denotes the set of possible types of player i. Let $\Omega \equiv \prod_{i \in N} \Omega_i$, $\Omega_{-i} \equiv \prod_{j \in N/\{i\}} \Omega_j$, $\omega = (\omega_i)_{i \in N} \in \Omega$, and $\omega_{-i} = (\omega_j)_{j \in N \setminus \{i\}} \in \Omega_{-i}$. The types in ω_i are distributed independently across players according to a probability measure in full support of Ω. Each player $i's$ payoff function satisfies quasi-linearity, risk-neutrality, and private values, that is, $v_i(a,\omega_i) - t_i$, where $t_i \in R$ denotes the monetary payment from player i to the central planner, and $v_i : A \times \Omega_i \to A$ is his valuation function over allocations.

A direct mechanism, or in short, a mechanism, is defined by (g,x), where $g : \Omega \to A$ denotes an allocation rule, $x : \Omega \to R^n$ denotes a payment rule, with $x = (x_i)_{i \in N}$ and $x_i : \Omega \to R$. According to (g,x), when each player $i \in N$ announces $\omega_i \in \Omega_i$, the allocation $g(\omega) \in A$ is selected and each player i makes a monetary payment of $x_i(\omega) \in R$ to the central planner, where $x(\omega) = (x_i(\omega))_{i \in N} \in R^n$.

We assume that the allocation rule g is *efficient* in the sense that for every $\omega \in \Omega$, the allocation $g(\omega) \in A$ maximizes the sum of all players' valuations:

$$\sum_{i \in N} v_i(g(\omega),\omega_i) = \max_{a \in A} \sum_{i \in N} v_i(a,\omega_i).$$

We focus on mechanisms that incentivize players to make honest announcements as Bayesian Nash equilibrium behavior, i.e., satisfies BIC.

Bayesian Incentive Compatibility (BIC): A mechanism (g,x) satisfies *BIC* if for every $i \in N$, $\omega_i \in \Omega_i$, and $\omega_i' \in \Omega_i$,

$$E[v_i(g(\omega),\omega_i) - x_i(\omega) \mid \omega_i] \geq E[v_i(g(\omega_i',\omega_{-i}),\omega_i) - x_i(\omega_i',\omega_{-i}) \mid \omega_i],$$

where $E[\cdot\,|\,\omega_i]$ denotes the interim expectation operator in terms of $\omega_{-i} \in \Omega_{-i}$ conditional on ω_i.

Let X denote the set of all payment rules x such that (g,x) satisfies BIC. Let $\hat{X} \subset X$ denote the set of all payment rules $x \in X$ such that there exist $h_i : \Omega_{-i} \to R$ for each $i \in N$ satisfying

$$x_i(\omega) = -\sum_{j \in N \setminus \{i\}} v_j(g(\omega), \omega_j) + h_i(\omega_{-i}) \text{ for all } \omega \in \Omega .$$

For each $x \in \hat{X}$, the mechanism (g,x) is a so-called Groves mechanism[6]. A Groves mechanism satisfies incentive compatibility in dominant strategy in the sense that for every $i \in N$, $\omega \in \Omega$, and $\omega_i' \in \Omega_i$,

$$v_i(g(\omega), \omega_i) - x_i(\omega) \geq v_i(g(\omega_i', \omega_{-i}), \omega_i) - x_i(\omega_i', \omega_{-i}) .$$

This paper assumes payoff equivalence in the sense that for every $x \in X$, there exists $\hat{x} \in \hat{X}$ that induces the same interim expected payments: for every $i \in N$,

$$E[\hat{x}_i(\omega)\,|\,\omega_i] = E[x_i(\omega)\,|\,\omega_i] \text{ for all } \omega_i \in \Omega_i .$$

The literature for mechanism design with side payments has shown sufficient conditions that guarantee payoff equivalence. See Williams (1999), Krishna and Maenner (2001), Krishna and Perry (2000), Milgrom and Segal (2002), and Bikhchandani et al. (2006), for instance.

The *revenue* for the central planner is defined as the sum of all players' payments, that is, $\sum_{i \in N} x_i(\omega)$. We assume that the central planner is *risk-averse*; the central planner prefers constant revenue across type profiles. Following Arrow (1979), d'Aspremont and Gérard-Varet (1979), and Krishna and Perry (2000), we can show that for every $\tilde{x} \in X$, there exists $x \in X$ that induces the same interim expected payoffs and a constant revenue that is the same as the ex-ante expected revenue induced by $(g,\tilde{x})$.

Lemma 1: *For every* $\tilde{x} \in X$ *, there exists* $x \in X$ *such that*

[6] See Groves (1973), Green and Laffont (1977), and Holmstrom (1979).

$$E[x_i(\omega) \mid \omega_i] = E[\tilde{x}_i(\omega) \mid \omega_i] \text{ for all } i \in N \text{ and } \omega_i \in \Omega_i ,$$

and

$$\sum_{i \in N} x_i(\tilde{\omega}) = E[\sum_{i \in N} \tilde{x}_i(\omega)] \text{ for all } \tilde{\omega} \in \Omega ,$$

where $E[\cdot]$ *denotes the expectation operator in terms of* $\omega \in \Omega$.

Proof: See the Appendix.

From Lemma 1, without loss of generality, we can confine our attention to payment rules $x \in X$ that are decomposed into $r = (r_i)_{i \in N} \in R^n$ and $y = (y_i)_{i \in N} \in X$, where

$$x_i(\omega) = y_i(\omega) + r_i \text{ for all } i \in N \text{ and all } \omega \in \Omega ,$$

and y satisfies *balanced budgets* in that

$$\sum_{i \in N} y_i(\omega) = 0 \text{ for all } \omega \in \Omega .$$

Note that $\sum_{i \in N} r_i$ corresponds to the central planner's revenue in the ex-post term, i.e.,

$$\sum_{i \in N} r_i = \sum_{i \in N} x_i(\omega) \text{ for all } \omega \in \Omega .$$

We will write (r, y) and $(g, (r, y))$ instead of x and (g, x), respectively.

For every $i \in N$ and $\omega_i \in \Omega_i$, let $U_i(\omega_i) \in R$ denote player i's *outside opportunity*. Let $U_N = (U_i)_{i \in N}$ denote the profile of outside opportunity functions, where $U_i : \Omega \to R$ for each $i \in N$. In order to make the allocation problem non-trivial, we assume that g induces a positive net ex-ante expected surplus such that

(1) $$E[\sum_{i \in N} v_i(g(\omega), \omega_i)] - E[\sum_{i \in N} U_i(\omega_i)] > 0 .$$

With the assumption of (1), without loss of generality, we can confine our attention to payment rules $x \in X$ that, along with the efficient allocation rule g, satisfy *ex-ante individual rationality* in the sense that for every $i \in N$,

$$E[v_i(g(\omega), \omega_i) - x_i(\omega)] \geq E[U_i(\omega_i)].$$

The following requirement, namely IIR, implies that the mechanism incentivizes each player not to exit from the allocation problem at the interim stage after observing his type.

Interim Individual Rationality (IIR): A mechanism (g,x) and a profile of outside opportunity functions U_N satisfy IIR if for every $i \in N$ and $\omega_i \in \Omega_i$,

$$E[v_i(g(\omega),\omega_i) - x_i(\omega) \mid \omega_i] \geq U_i(\omega_i).$$

Let us denote by $X(U_N) \subset X$ the set of all payment rules $x \in X$ such that (g,x) and U_N satisfy IIR. Let us denote by $\pi_0(U_N) \in R$ the *maximum revenue* defined by

$$\pi_0(U_N) \equiv \max_{x \in X(U_N)} E[\sum_{i \in N} x_i(\omega)].$$

From the above observations, it is clear that there exists $x = (r, y) \in X(U_N)$ such that the central planner's revenue as the sum of participation fees is equal to the maximal revenue:

$$\sum_{i \in N} r_i = \pi_0(U_N), \text{ that is, } \sum_{i \in N} x_i(\omega) = \pi_0(U_N) \text{ for all } \omega \in \Omega.$$

Proposition 2: *It holds that*

(2) $$\pi_0(U_N) = -(n-1)E[\sum_{i \in N} v_i(g(\omega),\omega_i)]$$
$$-\sum_{i \in N} \max_{\omega_i \in \Omega_i}\{U_i(\omega_i) - E[\sum_{j \in N} v_j(g(\omega),\omega_j) \mid \omega_i]\}.$$

Proof: Let us consider an arbitrary payment rule $x \in X(U_N)$, where for every $i \in N$ and $\omega_i \in \Omega_i$,

$$E[v_i(g(\omega)),\omega_i) - x_i(\omega) \mid \omega_i]$$
$$= E[v_i(g(\omega),\omega_j) + \sum_{j \in N\setminus\{i\}} v_j(g(\omega),\omega_j) - h_i(\omega_{-i}) \mid \omega_i]$$
$$= E[\sum_{j \in N} v_j(g(\omega),\omega_j) \mid \omega_i] - E[h_i(\omega_{-i})].$$

The inequalities in IIR are equivalent to the inequalities given by

$$-E[h_i(\omega_{-i})] \geq \max_{\omega_i \in \Omega_i}\{U_i(\omega_i) - E[\sum_{j \in N} v_j(g(\omega), \omega_j) \mid \omega_i]\}.$$

Hence,

$$E[x_i(\omega) \mid \omega_i] \leq -E[\sum_{j \in N \setminus \{i\}} v_j(g(\omega), \omega_j) \mid \omega_i]$$

$$-\max_{\omega_i \in \Omega_i}\{U_i(\omega_i) - E[\sum_{j \in N} v_j(g(\omega), \omega_j) \mid \omega_i]\} \quad \text{for all} \quad \omega_i \in \Omega_i,$$

that is,

$$E[x_i(\omega)] \leq -E[\sum_{j \in N \setminus \{i\}} v_j(g(\omega), \omega_j)]$$

$$-\max_{\omega_i \in \Omega_i}\{U_i(\omega_i) - E[\sum_{j \in N} v_j(g(\omega), \omega_j) \mid \omega_i]\}.$$

This implies that

$$E[\sum_{i \in N} x_i(\omega)] \leq -(n-1)E[\sum_{i \in N} v_i(g(\omega), \omega_i)]$$

$$-\sum_{i \in N} \max_{\omega_i \in \Omega_i}\{U_i(\omega_i) - E[\sum_{j \in N} v_j(g(\omega), \omega_j) \mid \omega_i]\}.$$

Hence,

$$\pi_0(U_N) \leq -(n-1)E[\sum_{i \in N} v_i(g(\omega), \omega_i)]$$

$$-\sum_{i \in N} \max_{\omega_i \in \Omega_i}\{U_i(\omega_i) - E[\sum_{j \in N} v_j(g(\omega), \omega_j) \mid \omega_i]\}.$$

For every $i \in N$, let us specify x in a manner that for every $\omega \in \Omega$,

$$x_i(\omega) = -\sum_{j \in N \setminus \{i\}} v_j(g(\omega), \omega_j)$$

$$-\max_{\omega_i \in \Omega_i}\{U_i(\omega_i) - E[\sum_{j \in N} v_j(g(\omega), \omega_j) \mid \omega_i]\} h_i(\omega_{-i}).$$

Clearly, the specified x satisfies IIR, and

$$E[\sum_{i \in N} x_i(\omega)] = -(n-1)E[\sum_{i \in N} v_i(g(\omega), \omega_i)]$$

$$-\sum_{i \in N} \max_{\omega_i \in \Omega_i}\{U_i(\omega_i) - E[\sum_{j \in N} v_j(g(\omega), \omega_j) \mid \omega_i]\},$$

which implies (2).

Q.E.D.

Provided that a payment rule $x = (r, y) \in X(U_N)$ induces the maximal revenue, that is, $\sum_{i \in N} r_i = \pi_0(U_N)$, let us denote $\pi_i(U_N) \in R$ as the corresponding ex-ante expected payoff for each player $i \in N$:

(3) $$\pi_i(U_N) = E[\sum_{i \in N} v_i(g(\omega), \omega_i)] + \max_{\omega_i \in \Omega_i}\{U_i(\omega_i) - E[\sum_{j \in N} v_j(g(\omega), \omega_j) \mid \omega_i]\}.$$

3. Combinatorial Allocations

Let us consider a combinatorial allocation problem, where the players and the central planner possess their respective initial endowments and trade these commodities simultaneously. There exist L heterogeneous commodities. We define an allocation as a profile of packages $a=(a_i)_{i\in N}$, where for every $i\in N$,

$$a_i \subset L, \text{ and } a_i \cap a_j = \phi \text{ for all } j \in N \setminus \{i\},$$

and $a_i \subset L$ denotes the package assigned to player i. Let $g(\omega)=(g_i(\omega))_{i\in N}\in A$. We assume that the valuation $v_i(a,\omega_i)$ depends just on (a_i,ω_i); we will write $v_i(a_i,\omega_i)$ instead of $v_i(a,\omega_i)$.

Let $e_i \subset L$ denote the *initial endowment* for each player $i\in N$, where

$$e_i \cap e_j = \phi \text{ for all } j \in N \setminus \{i\}.$$

The profile of initial endowments is regarded as an allocation $e\equiv(e_i)_{i\in N}\in A$. Let $e_0 \equiv L \setminus \bigcup_{i\in N} e_i$ denote the initial endowment of the central planner. We assume that the central planner has zero valuation for any package. We assume that each player can consume his initial endowment by himself whenever he exits from the allocation problem:

(4) $$U_i(\omega_i)=v_i(e_i,\omega_i) \text{ for all } i\in N \text{ and all } \omega_i\in\Omega_i.$$

For every subset of players, that is, every *coalition* $S\subset N$, we define $A(S)\subset A$ as the set of all allocations $a\in A$ such that

$$a_i = e_i \text{ for all } i\in N\setminus S.$$

Let us specify a function $g^S=(g_i^S)_{i\in S}:\Omega_S\to A(S)$ as the allocation rule that is *efficient for the coalition* $S\subset N$ in that for every $\omega_S\in\Omega_S$,

$$\sum_{i\in S} v_i(g_i^S(\omega_S),\omega_i)=\max_{a\in A(S)}\sum_{i\in S} v_i(a_i,\omega_i),$$

where $\Omega_S\equiv\prod_{i\in S}\Omega_i$ and $\omega_S\equiv(\omega_i)_{i\in S}\in\Omega_S$.

Proposition 3: *It holds that*

$$\pi_0(U_N) \geq -(n-1)E[\sum_{i\in N} v_i(g_i(\omega),\omega_i)] + E[\sum_{i\in N}\sum_{j\in N\setminus\{i\}} v_j(g_j^{N\setminus\{i\}}(\omega_{-i}),\omega_j)].$$

and for each $i \in N$,

$$\pi_i(U_N) \leq E[\sum_{j\in N} v_j(g_j(\omega),\omega_j)] - E[\sum_{j\in N\setminus\{i\}} v_j(g_j^{N\setminus\{i\}}(\omega_{-i}),\omega_j)].$$

Proof: From the definition of $g^{N\setminus\{i\}}$ and (4), it follows that for every $\omega_i \in \Omega_i$,

$$U_i(\omega_i) - E[\sum_{j\in N} v_j(g_j(\omega),\omega_j) \mid \omega_i]$$

$$\leq v_i(e_i,\omega_i) - E[v_i(e_i,\omega_i) + \max_{a\in A(\{i\})}\sum_{i\in N\setminus\{i\}} v_j(a_j,\omega_j) \mid \omega_i]$$

$$= -E[\sum_{j\in N\setminus\{i\}} v_j(g_j^{N\setminus\{i\}}(\omega_{-i}),\omega_j) \mid \omega_i],$$

which, along with (2) and (3), implies the proposition.

Q.E.D.

We define the *coalitional game* $\varpi : 2^N \setminus \{\phi\} \to R$ as assigning to each coalition $S \in 2^N \setminus \{\phi\}$ the maximal ex-ante expected surplus for the coalition S, that is,

$$\varpi(S) \equiv E[\sum_{j\in S} v_j(g^S(\omega_S),\omega_j)] \text{ for all } S \in 2^N \setminus \{\phi\}.$$

Note that

$$\varpi(N) \equiv E[\sum_{j\in N} v_j(g(\omega),\omega_j)].$$

It is clear from Proposition 3 that

(5) $$\pi_0(U_N) \geq -(n-1)\varpi(N) + \sum_{i\in N}\varpi(N\setminus\{i\}),$$

and for every $i \in N$,

(6) $$\pi_i(U_N) \leq \varpi(N) - \varpi(N\setminus\{i\}).$$

For every $i \in N$, a type $\omega_i^* \in \Omega_i$ is said to be an *opt-out type* if

$$g_i(\omega_i^*,\omega_{-i}) = e_i \text{ for all } \omega_{-i} \in \Omega_{-i}.$$

When player i has the opt-out type, the consumption of his initial endowment e_i by himself is valuable to the point that the efficient allocation rule g assigns it to him irrespective of the other players' types. The notion of the opt-out type was introduced by Makowski and Ostroy (1989). See also Segal and Whinston (2010). The assumption of the presence of opt-out types excludes near-equal share ownerships investigated by Cramton, Gibbons, and Klemperer (1987), which guarantees the non-negativity of revenue in bilateral trades.

Whenever there is an opt-out type in the set of possible types Ω_i for each player i, we can then replace Proposition 3 with the following characterization result.

Theorem 4: *Whenever there exists an opt-out type* $\omega_i^* \in \Omega_i$ *for each player* $i \in N$, *then*

(7) $$\pi_0(U_N) = -(n-1)\varpi(N) + \sum_{i \in N} \varpi(N \setminus \{i\}),$$

and for every $i \in N$,

(8) $$\pi_i(U_N) = \varpi(N) - \varpi(N \setminus \{i\}).$$

Proof: From the definition of opt-out type $\omega_i^* \in \Omega_i$,

$$E[\sum_{j \in N} v_j(g_j(\omega), \omega_j) \mid \omega_i^*] = E[\sum_{j \in N} v_j(g_j^{N\setminus\{i\}}(\omega_{-i}), \omega_j)].$$

Hence, in the same manner as the proof of Proposition 3,

$$\max_{\omega_i \in \Omega_i}\{U_i(\omega_i) - E[\sum_{j \in N} v_j(g_j(\omega), \omega_j) \mid \omega_i]\}$$

$$= U_i(\omega_i^*) - E[\sum_{j \in N} v_j(g_j(\omega), \omega_j) \mid \omega_i^*]$$

$$= -E[\sum_{j \in N\setminus\{i\}} v_j(g_j^{N\setminus\{i\}}(\omega_{-i}), \omega_j)].$$

This, along with (2) and (3), implies that

$$\pi_0(U_N) = -(n-1)E[\sum_{i \in N} v_i(g_i(\omega), \omega_i)] + E[\sum_{i \in N} \sum_{j \in N\setminus\{i\}} v_j(g_j^{N\setminus\{i\}}(\omega_{-i}), \omega_j)].$$

and for every $i \in N$,

$$\pi_i(U_N) = E[\sum_{i \in N} v_i(g_i(\omega), \omega_i)] - E[\sum_{j \in N\setminus\{i\}} v_j(g_j^{N\setminus\{i\}}(\omega_{-i}), \omega_j)].$$

These equalities, along with the definition of the coalitional game, imply (7) and (8)

Q.E.D.

We introduce *the first collective decision procedure* as follows. The central planner designs an efficient mechanism with BIC at the ex-ante stage, and at the interim stage, after observing his type, each player has the option to participate in or exit from the procedure. In order to incentivize each player to participate, the mechanism requires IIR. The equality (7) in Theorem 4 implies that with the presence of opt-out types, the central planner has a deficit in revenue if and only if

$$(n-1)\varpi(N) > \sum_{i\in N}\varpi(N\setminus\{i\}) .$$

4. Marginal Stability

This section introduces *the second collective decision procedure* as follows. At the ex-ante stage, each player i pays the participation fee r_i to the central planner. Each player commits himself not to exit from the procedure even after observing his type. The players then design the efficient mechanism $(g,y)\in X$ with BIC and with balanced budgets in the sense that

$$\sum_{i\in N} y_i(\omega)=0 \text{ for all } \omega\in\Omega .$$

Due to their commitment, the mechanism does not require IIR. From (1), without loss of generality, we can assume that $(g,(r,y))$ satisfies ex-ante individual rationality and *positive* revenue.

We assume that at the ex-ante stage, before the players design the mechanism (g,y), any size-($n-1$) coalition has the option to exclude the player who does not belong to this coalition from the allocation problem and occupy the entire set of commodities up for sale, including the central planner's endowment, except the excluded player's initial endowment. We require the mechanism to incentivize any size-($n-1$) coalition to never exercise this option, that is, to satisfy MS in the following sense.

Marginal Stability (MS): A mechanism (g,y) satisfies MS if for every $i\in N$,

(9) $$E[\sum_{j\in N\setminus\{i\}}\{v_j(g_j(\omega),\omega_j)-y_j(\omega)\}]\geq\varpi(N\setminus\{i\}),$$

where

$$\varpi(N\setminus\{i\})=E[\sum_{j\in N\setminus\{i\}}v_j(g_j^{N\setminus\{i\}}(\omega),\omega_j)].$$

Proposition 5: *There exist* $(r,y)\in X$ *, such that* $(g,(r,y))$ *satisfies MS if and only if*

(10) $$(n-1)\varpi(N)\geq\sum_{i\in N}\varpi(N\setminus\{i\}).$$

Proof: From the definition of $\varpi(N)$ and the balanced budgets of y,

$$\sum_{i\in N} E[\sum_{j\in N\setminus\{i\}} \{v_j(g_j(\omega),\omega_j) - y_j(\omega)\}] = (n-1)\varpi(N) ,$$

which along with (9), implies (10).

Suppose that (10) holds. Then, clearly, there exists $\tilde{\pi} = (\tilde{\pi}_i)_{i\in N} \in R^n$ satisfying that

(11) $$\sum_{j\in N\setminus\{i\}} \tilde{\pi}_j = \varpi(N\setminus\{i\}) \text{ for all } i \in N .$$

Let us specify $\pi = (\pi_i)_{i\in N} \in R^n$ by

$$\pi_i = \tilde{\pi}_i + \frac{\varpi(N) - \sum_{j\in N} \tilde{\pi}_j}{n} \text{ for all } i \in N .$$

Note that

$$\sum_{i\in N} \pi_i = \varpi(N) .$$

From (10) and (11),

$$\varpi(N) - \sum_{j\in N} \tilde{\pi}_j > 0 ,$$

and therefore,

(12) $$\pi_i > \tilde{\pi}_i \text{ for all } i \in N .$$

Since $\sum_{i\in N} \pi_i = \varpi(N)$, there exists a budget-balanced payment rule $y \in X$, such that

$$\pi_i = E[v_i(g_i(\omega),\omega_i) - y_i(\omega)],$$

which, along with (10), (11), and (12), implies (9) for all $i \in N$. Hence, $(g,(r,y))$ is marginally stable.

Q.E.D.

The following theorem shows that whenever there is an opt-out type for each player, then the existence of a marginally stable mechanism is equivalent to the central planner's deficit in maximal revenue.

Theorem 6: *If $\pi_0(U_N) \le 0$, then there exists a mechanism that satisfies efficiency, BIC, and MS. Whenever there is an opt-out type for each player, then there exists a mechanism that satisfies efficiency, BIC, and MS if and only if $\pi_0(U_N) \le 0$.*

Proof: From (5), it is evident that $\pi_0(U_N) \leq 0$ implies (10). From (7), it is evident that on the assumption that there is an opt-out type for each player, $\pi_0(U_N) \leq 0$ is equivalent to (10). This observation, in conjunction with Proposition 5, implies this theorem.

Q.E.D.

From Theorem 4, in the first collective procedure, it is inevitable for the central planner to have a deficit in revenue if and only if inequality (10) holds. From Theorem 6, in the second collective procedure, the central planner can earn a positive revenue if and only if inequality (10) holds. Hence, we can conclude that the first collective decision procedure fails to make the achievement of efficiency compatible with the central planner's non-negative revenue if and only if the second collective decision procedure succeeds to do the same.

5. Discussion: Standard Stability

The success of the second collective decision procedure might be in doubt if the players are permitted to have the option to build a coalition of any size. A mechanism (g, y) is said to satisfy *standard stability* (SS) if for every non-empty coalition $S \subset N$,

(11) $$E[\sum_{i \in S}\{v_i(g_i(\omega), \omega_i) - y_i(\omega)\}] \geq \varpi(S).$$

Following Milgrom (2004, Chapter 8), for instance, whenever the commodities are substitutes, then any Groves mechanism, which generally satisfies MS, automatically satisfies SS. However, whenever the commodities are not necessarily substitute, then, SS is generally more restrictive than MS. Hence, by replacing MS with SS, it might be the case that both of the first and second collective decision procedures fail to achieve efficiency in a consistent manner with the central planner's non-negative revenue.

Let us consider the combinatorial allocation problem, in which the underlying coalitional game has a *symmetric* structure in a manner that for every $S \subset N$ and every $S' \subset N$,

$$\varpi(S) = \varpi(S') \qquad \text{if } |S| = |S'|.$$

With this symmetry, we will write $\varpi(l)$ instead of $\varpi(S)$, where $l = |S|$. In the same manner as Proposition 5, it follows that there exists $(r, y) \in X$ such that $(g, (r, y))$ satisfies SS if and only if for every $l \in \{1, ..., n-1\}$,

$$l\varpi(n) > n\varpi(l),$$

where we design (r, y) so that it induces the same ex-ante expected payoff across players.

Suppose that

$$(n-1)\varpi(n) > n\varpi(n-1),$$

while there exists $l \in \{1, ..., n-2\}$ such that

$$l\varpi(n) \leq n\varpi(l).$$

Then, from Theorem 5 and the above-mentioned observations, the first and second collective decision procedures both fail to make the achievement of efficiency consistent with the central planner's non-negative revenue.

In order to overcome this difficulty, the central planner should change the commodities brought by him in a manner that crucially depends on which procedure between the first and second collective decision procedures to follow.

In the first collective procedure, the central planner should change the commodities brought by him in order to make the coalitional game satisfy

$$(n-1)\varpi(n) \leq n\varpi(n-1).$$

In this case, the central planner should bring more commodities that are valuable to the players compared to their initial endowments. By doing this, the central planner can reduce the bargaining rents of the players, thereby increasing his revenue.

In the second collective procedure, the central planner should change the commodities brought by him in order to make the coalitional game satisfy

$$l\varpi(n) > n\varpi(l).$$

In this case, the central planner should refrain from bringing commodities that are valuable to the players. By doing this, the central planner can enhance the bargaining rents of the players, thereby making the collective decision more stable.

6. Conclusion

This paper investigated the combinatorial allocation problem with incomplete information, where both the central planner and the players brought commodities to sell simultaneously. The central planner attempted to achieve efficiency in the incentive compatible manner, without having a deficit in revenue.

We introduced two distinct collective decision procedures. The first procedure assumed that the central planner had the initiative in designing a mechanism, and each player had the option to exit from the procedure after observing his type. This procedure required interim individual rationality. In contrast, the second procedure assumed that the central planner transferred the initiative of mechanism design to the players, who have to commit themselves to participate in the procedure. This procedure required stability against blocking behavior by the largest proper coalitions, that is, MS.

With the presence of opt-out types, we showed the characterization result that the central planner could earn non-negative revenue in the first procedure if and only if he failed in the second procedure. In the first collective decision procedure, the central planner should bring more commodities to sell. In the second collective decision procedure, the central planner should refrain from bringing commodities to sell.

References

Arrow, K. (1979): The Property Rights Doctrine and Demand Revelation under Incomplete Information," in *Economics and Human Welfare*, ed. by M. Boskin. New York, Academic Press.

Ausubel, L. and P. Milgrom (2002): "Ascending Auctions with Package Bidding," *Frontier of Theoretical Economics* 1, 1–42.

Bikhchandani, S., S. Chatterji, R. Lavi, A. Mu'alem, N. Nisan, and A. Sen (2006): "Weak Monotonicity Characterizes Deterministic Dominant-Strategy Implementation," *Econometrica* 74, 1109–1132.

Cramton, P. (2011): "Incentive Auctions and Spectrum Policy," Working Paper, University of Maryland.

Cramton, P., R. Gibbons, and P. Klemperer (1987): "Dissolving a Partnership Efficiently," *Econometrica* 55, 615–632.

Cramton, P., Y. Shoham, and R. Steinberg (2006): *Combinatorial Auctions*, MIT Press: Cambridge.

D'Aspremont, C. and L. Gérard-Varet (1979): "Incentive and Incomplete Information," *Journal of Public Economics* 11, 25-45.

Green, J. and J.-J. Laffont (1977): "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods," *Econometrica* 45, 427–438.

Groves, T. (1973): "Incentives in Teams," *Econometrica* 61, 617–631.

Hazlett, T., D. Porter, and V. Smith (2012): "Incentive Auctions: Economic and Strategic Issues," mimeo.

Holmstrom, B. (1979): "Groves Schemes on Restricted Domains," *Econometrica* 47, 1137–1144.

Kelso, A. and V. Crawford (1982): "Job Matching, Coalition Formation, and Gross Substitutes," *Econometrica* 50, 1483.

Krishna, V. (2010): *Auction Theory*, Second Edition, New York, Academic Press.

Krishna, V. and E. Maenner (2001): "Convex Potentials with an Application to Mechanism Design," *Econometrica* 69, 1113–1119.

Krishna, V. and M. Perry (2000): "Efficient Mechanism Design," mimeo, Penn. State

University.

Makowski, L. and J. Ostroy (1989): “Efficient and Individually Rational Bayesian Mechanisms Only Exist on Perfectly Competitive Environments,” UCLE Department of Economics, Discussion Paper 566.

Matsushima, H. (2011): “Efficient Combinatorial Exchange,” University of Tokyo, Faculty of Economics, Discussion Paper CIRJE-F-826.

Milgrom, P. (2004): *Putting Auction Theory to Work*, Cambridge, UK: Cambridge University Press.

Milgrom, P. (2007): “Package Auctions and Exchanges,” *Econometrica* 75, 935–965.

Milgrom, P., L. Ausubel, J. Levin, and I. Segal (2012): “Incentive Auction Rules Option and Discussion,” mimeo.

Milgrom, P. and I. Segal (2002): “Envelope Theorem for Arbitrary Choice Space,” *Econometrica* 70, 583–601.

Myerson, R. and M. Satterthwaite (1983): “Efficient Mechanisms for Bilateral Trading,” *Journal of Economic Theory* 29, 265–281.

Rassenti, S., R. Bulfin, and V. Smith (1982): A Combinatorial Auction Mechanism for Airport Time Slot Allocation,” *Bell Journal of Economics* 13, 402–417.

Segal, I. and M. Whinston (2010): “Property Rights,” forthcoming in *Handbook of Organizational Economics*, ed. by R. Gibons and J. Roberts, Princeton University Press: Princeton.

Williams, S. (1999): “A Characterization of Efficient, Bayesian Incentive Compatible Mechanisms,” *Economic Theory* 14, 155-180.

Appendix: Proof of Lemma 1

From payoff equivalence, without loss of generality, we can assume $\tilde{x} \in \hat{X}$. According to Arrow (1979) and d'Aspremont and Gérard-Varet (1979), it is evident that there exists $x' \in X$ such that (g, x') satisfies BIC and the balanced budgets in the sense that

$$\sum_{i \in N} x_i'(\omega) = 0 \quad \text{for all} \quad \omega \in \Omega .$$

From payoff equivalence, it is evident that there exists $(b_i)_{i \in N} \in R^n$ such that

$$E[\tilde{x}_i(\omega) \mid \omega_i] = E[x_i'(\omega) \mid \omega_i] + b_i \quad \text{for all} \quad i \in N \quad \text{and all} \quad \omega_i \in \Omega_i ,$$

where note that

$$\sum_{i \in N} b_i = E[\sum_{i \in N} \tilde{x}_i(\omega)] .$$

We specify $x \in X$ by

$$x_i(\omega) = x_i'(\omega) + b_i \quad \text{for all} \quad i \in N \quad \text{and all} \quad \omega \in \Omega .$$

From this specification, it is clear that (g, x) satisfies BIC,

$$E[x_i(\omega) \mid \omega_i] = E[x_i'(\omega) \mid \omega_i] + b_i$$

$$= E[\tilde{x}_i(\omega) \mid \omega_i] \quad \text{for all} \quad i \in N \quad \text{and} \quad \omega_i \in \Omega_i ,$$

and

$$\sum_{i \in N} x_i(\omega) = \sum_{i \in N} x_i'(\omega) + \sum_{i \in N} b_i = E[\sum_{i \in N} \tilde{x}_i(\omega)] \quad \text{for all} \quad \omega \in \Omega .$$