Role of Leverage in Bubbles and crashes

Hitoshi Matsushima

University of Tokyo

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Strategic Foundation for Bubbles and Crashes in Stock Market

Preemption (timing) game in finite time horizon \([0,1]\):
Professionals know bubble crashes certainly at or before terminal time 1
cf. Rational bubbles (Tirole (85))

With complete information:
Professional Arbitrageurs tail-chase compete to time market quickest
⇒ Bubble immediately crashes according to backward induction method

With some aspects of incomplete information:
They may stop competing and ride bubble

Abreu and Brunnermeier (03): Sequential awareness
Matsushima (12): Behavioral arbitrageurs, more tractable, standard

Both papers analyze only Harmless Bubble:
Company never raises and wastes funds during bubble
The present paper analyzes Harmful Bubble

Unproductive company raises and wastes huge funds during bubble
cf. Stagnation after bubble

Purpose:

Formulation of preemption game a la Matsushima (12) in which long persistence of bubble is unique Nash equilibrium

Main message:

High leverage allowed arbitrageurs encourage long persistence
⇒ Large social cost emerges despite tiny share price growth rate
Stock Market with Share Issuance and Leverage Ratio \( L \geq 1 \)

No interest rate, no dividend, no short selling

**Question:** Can bubble persist or immediately crash?
Positive feedback traders: Slaves of euphoria

Misperception per share

$P(0)$

$P(t)$

$P(1)$

$FV=0$

Initial time

$t$

Terminal time

Time
Company raises fund by issuing $S'(t)\Delta$ during bubble. However, selling pressure may dampen mispricing.

Company needs large patronage
$n \geq 0$ Arbitrageurs are not necessarily rational (Matsushima (12))

<table>
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<tr>
<th>Probability $\epsilon &gt; 0$:</th>
<th>Behavioral</th>
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<tbody>
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<td>Never time even at terminal time 1 (commit to ride bubble)</td>
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<th>Probability $1 - \epsilon &gt; 0$:</th>
<th>Rational</th>
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<td>Choice of timing Strategy: $q_i(t) \in [0,1]$</td>
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Each arbitrageur purchases $S_i'(t)\Delta$ during bubble: Short term debt contracts with naive traders a la Ponzi Scheme

Leverage ratio $L \geq 1$, Utilize stockholding $S_i(t)$ as collateral

Debt obligation

$$\frac{L-1}{L}S_i(t)P(t)$$

Personal capital

$$W_i(t) = S_i(t)P(t) - \frac{L-1}{L}S_i(t)P(t) = \frac{S_i(t)P(t)}{L}$$

∴ $W_i'(t) = \frac{S_i'(t)P(t) + S_i(t)P'(t)}{L}$

Capital gain

$S_i(t)\{P(t + \Delta) - P(t)\}$

∴ $W_i(t + \Delta) = W_i(t) + S_i(t)\{P(t + \Delta) - P(t)\}$

∴ $W_i'(t) = S_i(t)P'(t)$
We have:

\[ S_i(t)P'(t) = \frac{S'_i(t)P(t) + S_i(t)P'(t)}{L} \]

\[ \therefore \frac{S'_i(t)}{S_i(t)} = (L - 1) \frac{P'(t)}{P(t)} \]

Hence, if arbitrageur rides bubble at time \( t \in [0, 1] \), his stockholding must equal

\[ S_i(t) = S_i(0) \left( \frac{P(t)}{P(0)} \right)^{L-1} \]

**What kind of persons are naive traders?**

They incorrectly expects spot market price reflects FV, but not active like PFT.
PFTs hold \( S(t) - \sum_{i=1}^{n} S_i(t) \) in totality

However, selling pressure dampens their euphoria, bursting bubble

**Assumption:** Bubble crashes at or before time \( t \) if and only if

\[
\sum_{i=1}^{n} S_i(t) \leq (n - \tilde{n})cS(t)
\]

A rational arbitrageur sells up

\( \Rightarrow \) Other arbitrageurs, rational or behavioral, immediately sell up: Musical Chairs

\( \Rightarrow \) Bubble crashes, \( \tilde{n} \) arbitrageurs win capital gain

ex. Currency Attacks (Morris and Shin (98))
How should company make share issuance?

Company should raise fund up to the point in which arbitrageurs absorb $nc \times 100\%$ shares: otherwise bubble crashes

$$\therefore S(t) = \frac{S_i(t)}{c}$$

$$\therefore S(t) = S(0)\left(\frac{P(t)}{P(0)}\right)^{L-1}$$

With sufficient leverage ratio

Company’s market value extremely expands even if PFTs are not very enthusiastic, i.e., $\frac{P(t)}{P(0)}$ is close to 1
Arbitrageurs’ incentive to purchase shares

Win to time  \[ W_i(t) = \frac{S_i(t)P(t)}{L} \]

lose  \[ 0 \]

Hence, first order condition:

\[
(1 - \frac{\tilde{n} - 1}{n - 1})W_i(t) \frac{\partial}{\partial t} \{1 - D_i(t; q_{-i})\} + W_i'(t)\{1 - D_i(t; q_{-i})\} = 0
\]

\[
\therefore \frac{n - \tilde{n}}{n - 1} W_i(t) \frac{\partial D_i(t; q_{-i})}{\partial t} = W_i'(t)\{1 - D_i(t; q_{-i})\}
\]

where \(1 - D_i(t; q_{-i})\) denotes willing probability
Bubble-crash NE \( \tilde{q} \): Symmetric

\[
D(t; \tilde{q}) \equiv 1 - [1 - (1 - \varepsilon)\tilde{q}_1(t)]^\theta
\]

Hazard rate
\[
\theta(t) = L \frac{n}{n - \tilde{n}} \left( \frac{P'(t)}{P(t)} \right)
\]

Critical time \( \tilde{\tau} \):
\[
\varepsilon = \left( \frac{P(\tilde{\tau})}{P(1)} \right)^{\frac{L}{n - \tilde{n}}}
\]
Main Theorem:

Bubble-crash $\tilde{q}$ is NE if and only if $\varepsilon \geq \left( \frac{P(0)}{P(1)} \right)^{\frac{L}{n-\bar{n}}}$

Bubble-crash $\tilde{q}$ is unique NE if $\varepsilon > \left( \frac{P(0)}{P(1)} \right)^{\frac{L}{n-\bar{n}}}$

$$1 - \varepsilon \left( \frac{P(1)}{P(t)} \right)^{\frac{L}{n-\bar{n}}}$$

$$\tilde{q}_1(t) = \frac{1 - \varepsilon}{1 - \varepsilon}$$ for all $t \in [\tilde{\tau}, 1]$

$$\tilde{q}_1(t) = 0$$ for all $t \in [0, \tilde{\tau}]$

cf. $\varepsilon = 0$: Tail-chase competition immediately bursts bubble
Implications

With sufficient leverage ratio $L$:

Arbitrageurs could be expected to be rational almost certainly: $\varepsilon$ could be close to zero

PFTs are not very enthusiastic: $\frac{P(1)}{P(0)}$ could be close to zero

$\Rightarrow$ Despite them, bubble does persist for a long time
Bubble is harmful

**Social Cost** as total fund raised up to crash time \( t \)

\[
c(t) \equiv \int_{\tau=0}^{t} P(\tau)S'(\tau)d\tau = P(0)S(0)\frac{L-1}{L}\{(\frac{P(t)}{P(0)})^{L-1} - 1\}
\]

\[
\therefore \quad \frac{\text{Social Cost}}{\text{Market Value}} = \frac{C(t)}{P(t)S(t)} = \frac{L-1}{L} \frac{P(0)}{P(t)} \{1 - (\frac{P(0)}{P(t)})^{L-1}\}
\]

\[
\lim_{L \to \infty, \frac{P(0)}{P(t)} \uparrow 1} \frac{C(t)}{P(t)S(t)} = 1
\]

Small but long price bubble and extreme expansion imply ‘All Waste’
Two effects of Leverage

Promoting unproductive business expansion

Company attempts to issue lots of new shares but is afraid selling pressure dampens euphoria. Company expects arbitrageurs’ leveraged purchase to relieve this pressure.

Letting arbitrageurs prefer unproductive business expansion

Increase in capital gain is all arbitrageur’s profit because of debt holders’ naivety. Arbitrageur can let debt holders bear $\frac{L-1}{L} \times 100\%$ of loss by crash.