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Mechanism Design in Hidden Action and Hidden Information: Richness and Pure Groves

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1. Introduction

Mechanism Design with Side Payments: **Hidden action (Ex-Ante Investment)**
hidden information (Revelation)
ex. Auction, Public Goods, Principal-agent, Partnerships, …

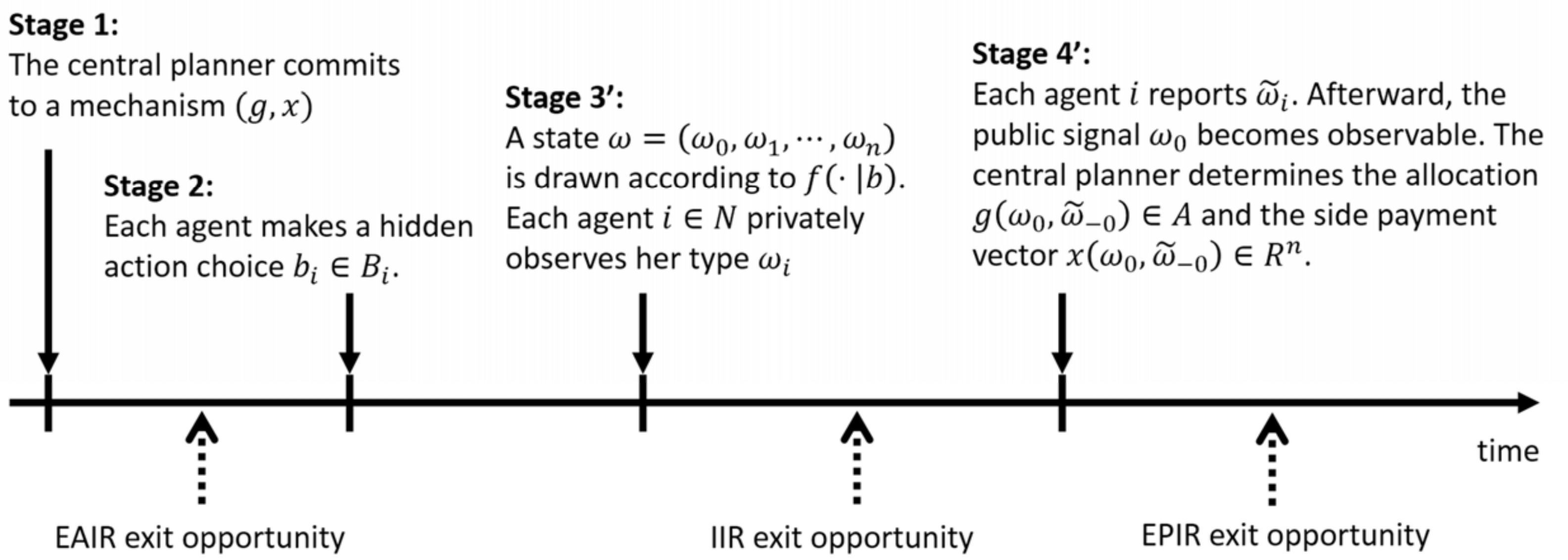
Timeline (General)

Stage 1 CP designs and commits to mechanism (g, x), $g : \Omega \to A$, $x = (x_i) : \Omega \to R^n$.
A denotes the set of alternatives, Ω denotes the state space.

Stage 2 (Hidden action)
Agents make action choices $b = (b_1, ..., b_n) \in B = \underset{i \in N}{\times} B_i$ at the expense of $c_i(b_i)$.

Stage 3' (Hidden information)
A state $\omega = (\omega_0, \omega_1, \omega_2, ..., \omega_n) \in \Omega$ occurs.
Action choices influences state distribution $f(\cdot \mid b) : \Omega \to [0,1]$.
Each agent $i \in N$ privately observes $\omega_i \in \Omega_i$ as his (or her) type.

Stage 4' Each agent reports $\tilde{\omega}_i \in \Omega_i$.
After their reports, CP and agents observe public signal $\omega_0 \in \Omega_0$.
CP determines $g(\omega_0, \tilde{\omega}_1, ..., \tilde{\omega}_n) \in A$ and $x(\omega_0, \tilde{\omega}_1, ..., \tilde{\omega}_n) \in R^n$.


Stage 1:
The central planner commits to a mechanism (g, x)
Stage 2:
Each agent makes a hidden action choice $b_i \in B_i$.
Stage 3′:
A state $\omega = (\omega_0, \omega_1, \cdots, \omega_n)$ is drawn according to $f(\cdot\,|b)$. Each agent $i \in N$ privately observes her type ω_i
Stage 4′:
Each agent i reports $\widetilde{\omega}_i$. Afterward, the public signal ω_0 becomes observable. The central planner determines the allocation $g(\omega_0, \widetilde{\omega}_{-0}) \in A$ and the side payment vector $x(\omega_0, \widetilde{\omega}_{-0}) \in R^n$.
time
EAIR exit opportunity
IIR exit opportunity
EPIR exit opportunity

Richness

(Key Assumption of This Paper)

Each agent has various aspects of activities.

information acquisition, R&D investment, patent control, standardization, M&A, rent-seeking, positive/negative campaigns, environmental concern, product differentiation, entry/exit decisions, preparation of infrastructure, headhunting……

Each agent's action choice has significant *externality effects*.

Each agent can change the state distribution, including the other agents' types, in various directions.

Question:

Can CP solve both incentives in hidden action and in hidden information ?

How?

To what degree?

Example:
Single-Unit Auction

Failure of Second-Price Auction (SPA)

Each bidder $i \in N$ announces price bid $m_i \geq 0$.
He (or she) obtains payoff

$\omega_i - \max_{j \neq i} m_j$ **if he wins, i.e., $g(m) = i$ (or $m_i > \max_{j \neq i} m_j$).**

0 **if he loses, i.e., $g(m) \neq i$ (or $m_i < \max_{j \neq i} m_j$).**

Truth-telling $m_i = \omega_i$ is a dominant strategy in SPA.
SPA solves incentive in hidden information, and achieves allocative efficiency.

What's wrong with SPA ?

SPA fails to achieve efficiency in hidden action.

Each bidder makes ex-ante investment that influences the other bidders' valuations.

In order to save the winner's payment $\max_{j \neq i} m_j = \max_{j \neq i} \omega_j$,

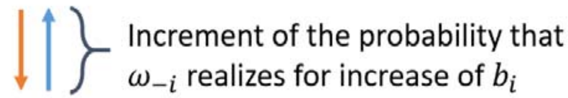
each bidder i **makes under-investment that decreases the others' valuations** ω_{-i}.

$\Rightarrow$ **SPA fails to achieve ex-ante efficiency in hidden action.**

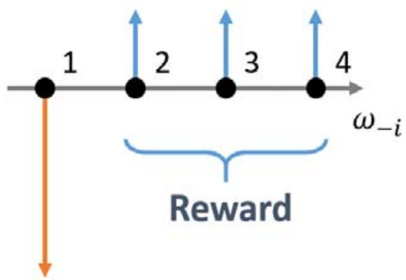
We need another protocol design!

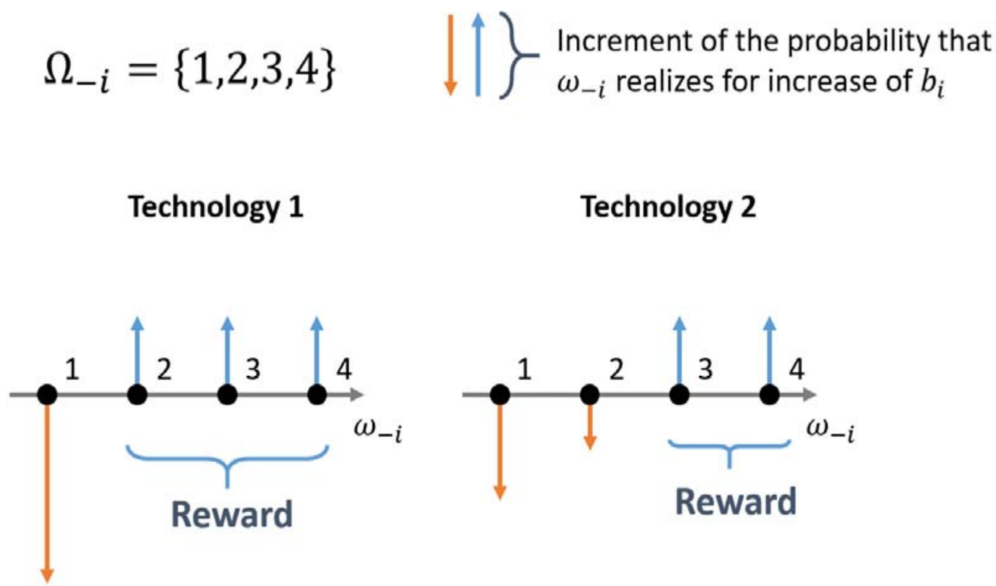
Each bidder has various technologies: ‘richness’ in this paper’s terminology

$$\Omega_{-i} = \{1,2,3,4\}$$

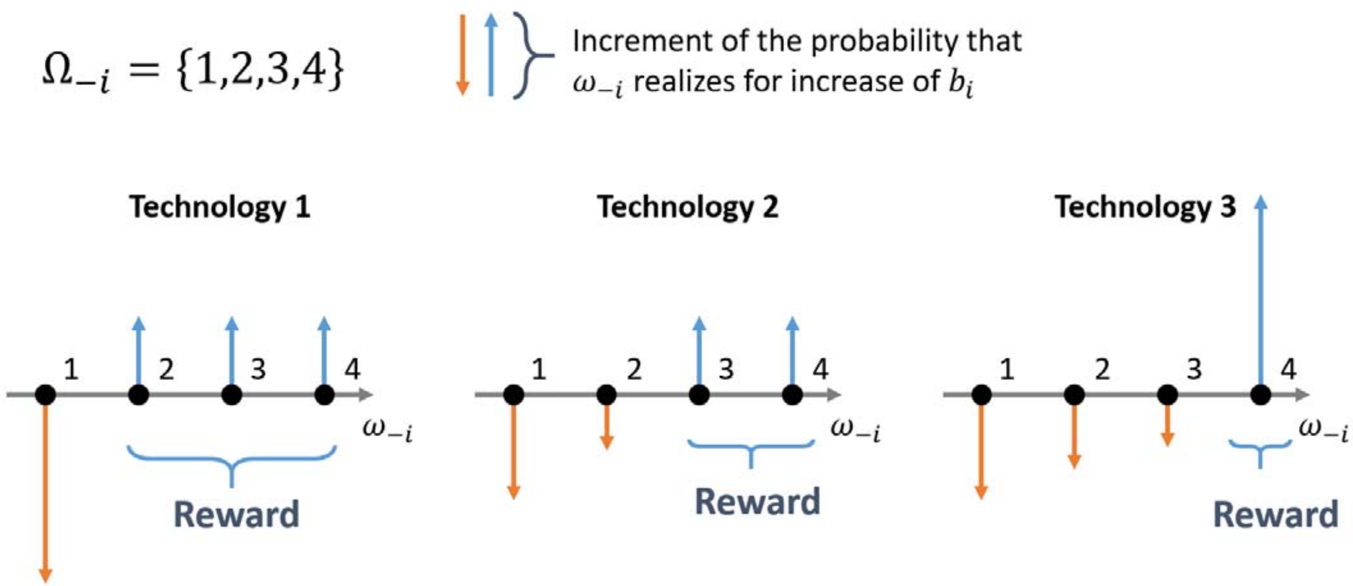


Technology 1

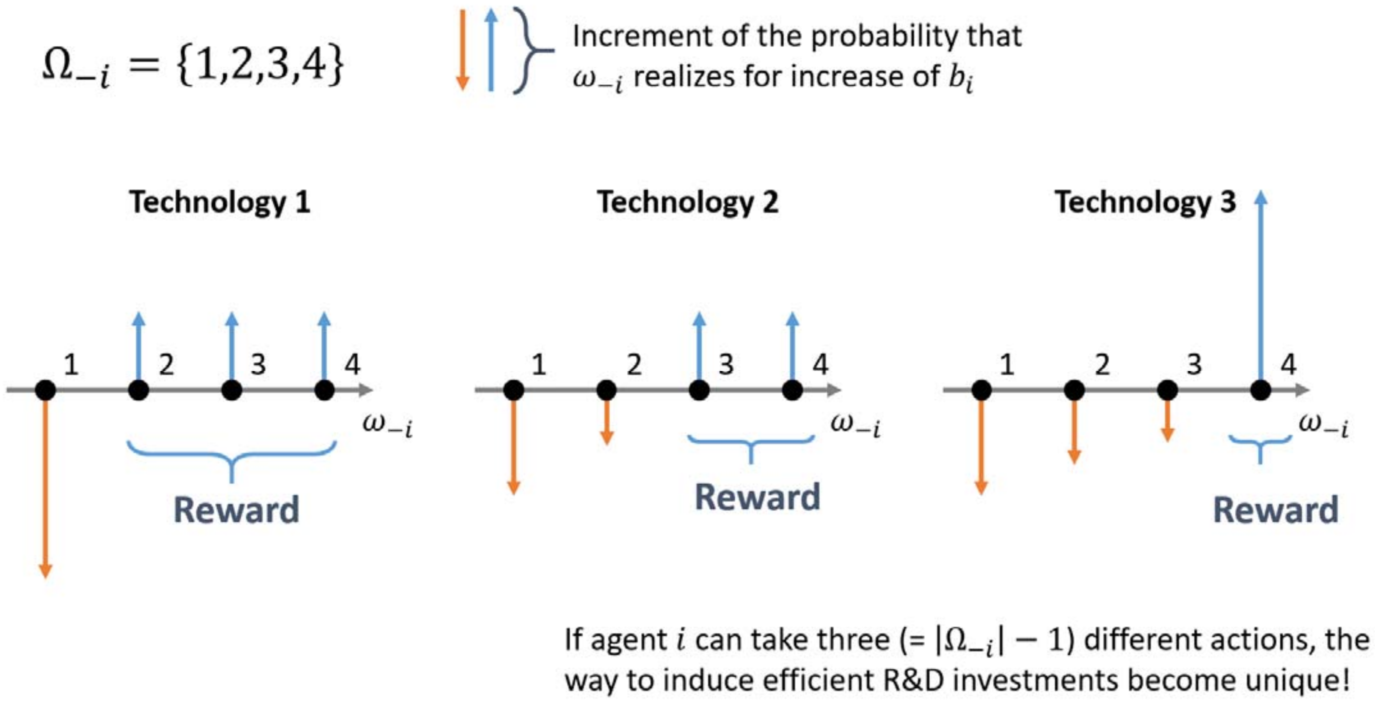




$\Omega_{-i} = \{1,2,3,4\}$
Increment of the probability that ω_{-i} realizes for increase of b_i
Technology 1
Technology 2
1
2
3
4
ω_{-i}
Reward
1
2
3
4
ω_{-i}
Reward


$\Omega_{-i} = \{1,2,3,4\}$
Increment of the probability that ω_{-i} realizes for increase of b_i
Technology 1
Technology 2
Technology 3
1
2
3
4
ω_{-i}
Reward

CP must take into account technologies 1, 2, and 3 altogether.



How should CP design mechanism?

Answer: Let's design 'Pure Groves' mechanism (PGM)!

What is Pure Groves Mechanism (PGM) ?
A Variant of Posted Price Scheme

CP fixes a price z_i for each bidder $i \in N$ in advance.

Each bidder $i \in N$ reports $m_i \geq 0$.

The winner i $(m_i \geq m_j, j \neq i)$ pays z_i to CP.

Each loser $j \neq i$ receives loser's gain $\max_{i \in N} m_i - z_j$ from CP.

Truth-telling is a dominant strategy in PGM, because PGM is Groves.

Pure Groves mechanism solves incentive not only in hidden information but also in hidden action

Each bidder is willing to make ex-ante investment in PGM,
Because it increases loser's gain $\max_{i \in N} m_i - z_i$**,**
while keeping the winner's payment z_i **unchanged.**

More Practice:

Symmetric Pure Groves Mechanism ($z_1 = z_i = z$ for all $i \in N$) is equivalent to 'Descending Auction for Determining Loser's Gain'

CP fixes (common) price z in advance.
CP conducts 'Descending Auction' for the determination of losers' gain.
First bidder who drops his hand becomes winner, getting commodity at price z.
The price level at which the winner drops his hand, i.e., $k \in R$, is regarded as loser's gain.
Each loser receives k from CP.

Dropping hands at the price level $\omega_i - z$ is a dominant strategy, achieving efficiency and the same payments as symmetric PG, hence solving both hidden action and hidden information.

However, in PGM,
Each bidder (low valuation) may have negative payoff in ex-post term.

Ex-Post Individual Rationality (EPIR) may be questionable.
We need ‘commitment device’ such as deposit requirement at the interim stage (stage 3’).

Fortunately, we can show
Interim Individual Rationally (IIR) is generally harmless.

End of SPA Example

Main Results of This Paper

Result I: Inducibility (Incentive in hidden action)

Assumption of Richness dramatically restricts the range of mechanisms that can induce the desired action profile as a NE outcome.

Ex-post Equivalence:	**Payments, revenue, and payoffs are unique up to constants.**
Pure Groves:	**A mechanism induces an efficient action profile if and only if it is pure Groves.**
Deficits and IR:	**There may exist no mechanism that satisfies non-negative expected revenue and ex-post individual rationality (EPIR).** **Commitment devices guarantees interim individual rationality (IIR).**

Result II. Incentive Compatibility (Incentive in hidden information, EPIC)

Any mechanism that solves hidden action automatically solves hidden information (EPIC).

Result III. No Externality (Private Richness)

Without externality, a much wider class of mechanisms, namely, 'expectation-Groves', solves both hidden action and hidden information without deficits, or with budget-balancing.

2. Related Literatures

Green and Laffont (1977, 1979), Holmström (1979):
Characterization of Groves from hidden information
cf. Characterization of pure Groves from hidden action

Bergemann and Valimaki (2002): **Private Values vs Interdependent Values**

Hatfield, Kojima, and Kominers (2015):
No externality, detail-freeness, Groves
cf. With and/or without richness, expectation-Groves

Obara (2008): **Mixed actions, unbounded side payments, approximate full surplus extraction**
cf. Bounded side payments, pure actions, deficits

Athey and Segal (2013): **Sufficiency of pure Groves**
cf. Necessity of pure Groves under richness

3. Benchmark
(with hidden Action, but without hidden information)

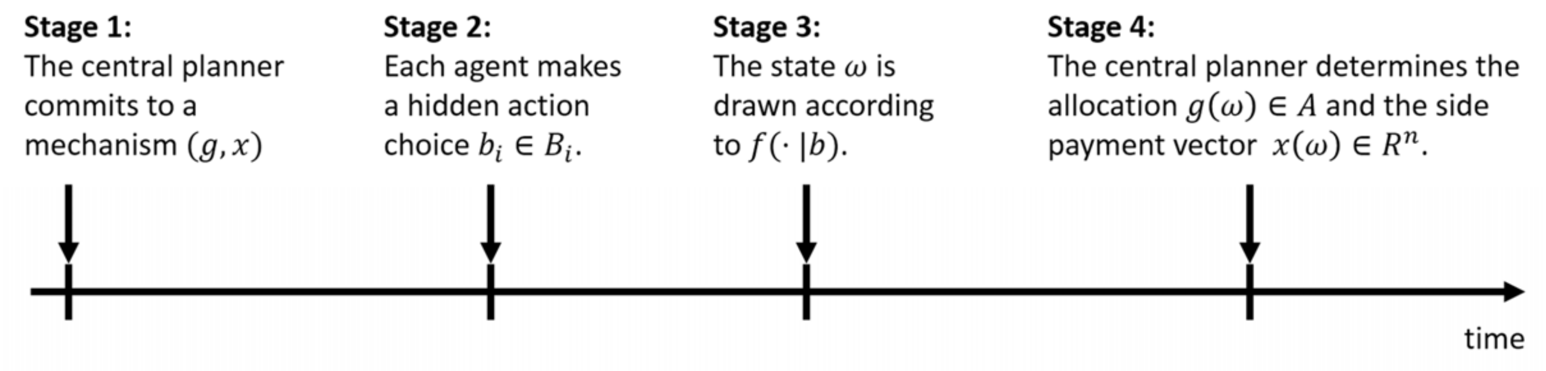
Timeline (without hidden information)

Stage 1: CP designs a mechanism (g, x).

Stage 2 (Hidden action) Agents make action choices $b \in B$ at the expense of $c_i(b_i)$.

Stage 3 (No-hidden-information) A state $\omega \in \Omega$ occurs.
CP and all agents observe ω.

Stage 4: CP determines $g(\omega) \in A$ and $t(\omega) \in R^n$.



Inducibility: Definition

A mechanism (g, x) is said to *induce* an action profile $b \in B$ if b is a NE, i.e.,

(2) $$E[v_i(g(\omega),\omega) - x_i(\omega) \mid b] - c_i(b_i) \geq E[v_i(g(\omega),\omega) - x_i(\omega) \mid b_i', b_{-i}] - c_i(b_i')$$

for all $i \in N$ and $b_i' \in B_i$.

Richness: Definition (1)

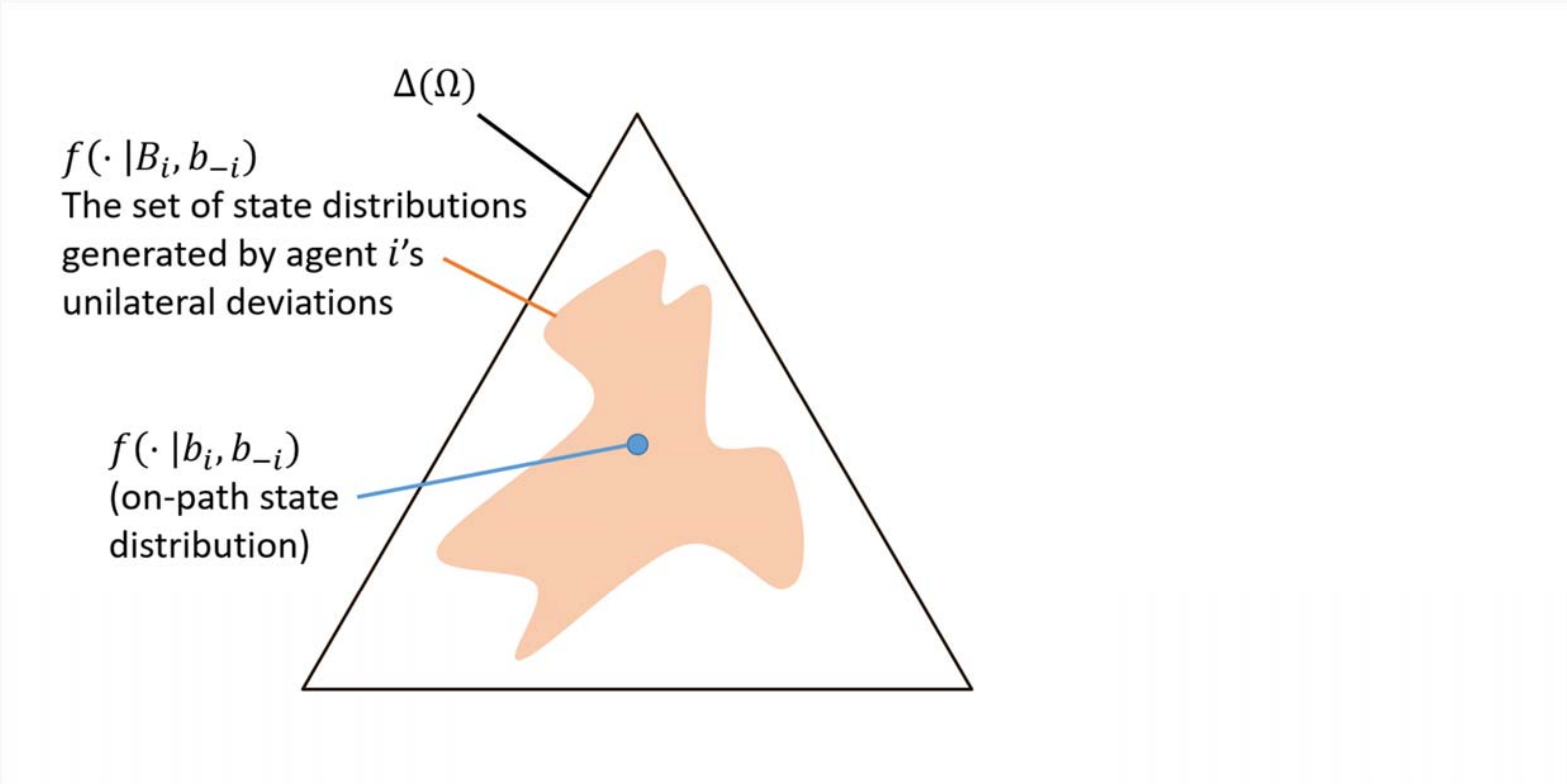
Each agent $i \in N$ can smoothly and locally change
the distribution of state in all directions from $f(\cdot \mid b)$
through pure action deviation.

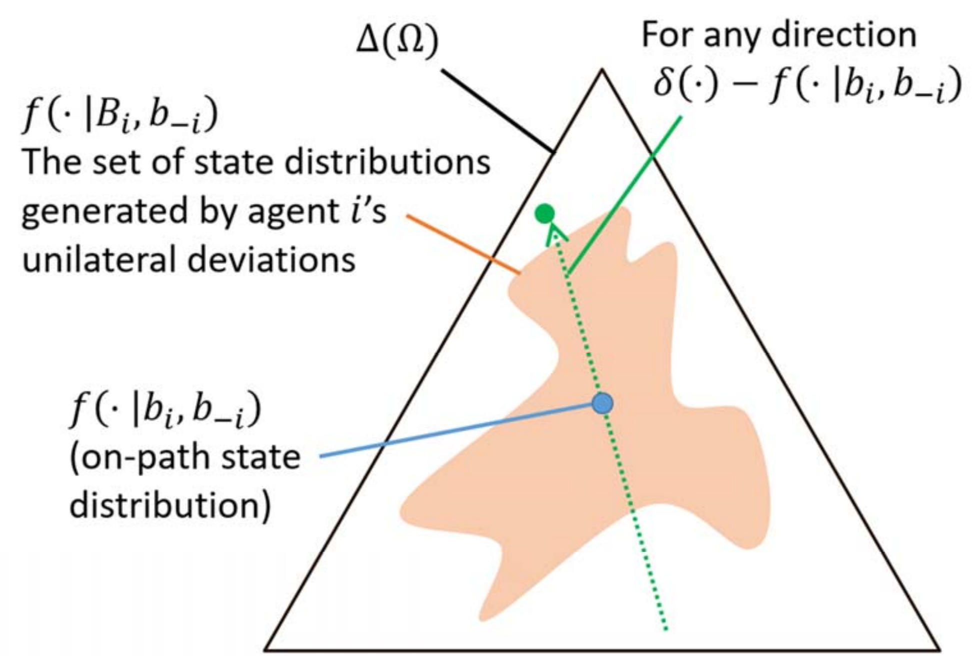
An action profile $b \in B$ is said to be *rich* if for every $i \in N$ and $\delta \in \Delta(\Omega)$, there exist $\bar{\alpha} > 0$ and a path on B_i , $\beta_i(\delta, \cdot) : [-\bar{\alpha}, \bar{\alpha}] \rightarrow B_i$, such that $\beta_i(\delta, 0) = b_i$,

(4) $$\lim_{\alpha \to 0} \frac{f(\cdot \mid \beta_i(\delta, \alpha), b_{-i}) - f(\cdot \mid b)}{\alpha} = \delta(\cdot) - f(\cdot \mid b) ,$$

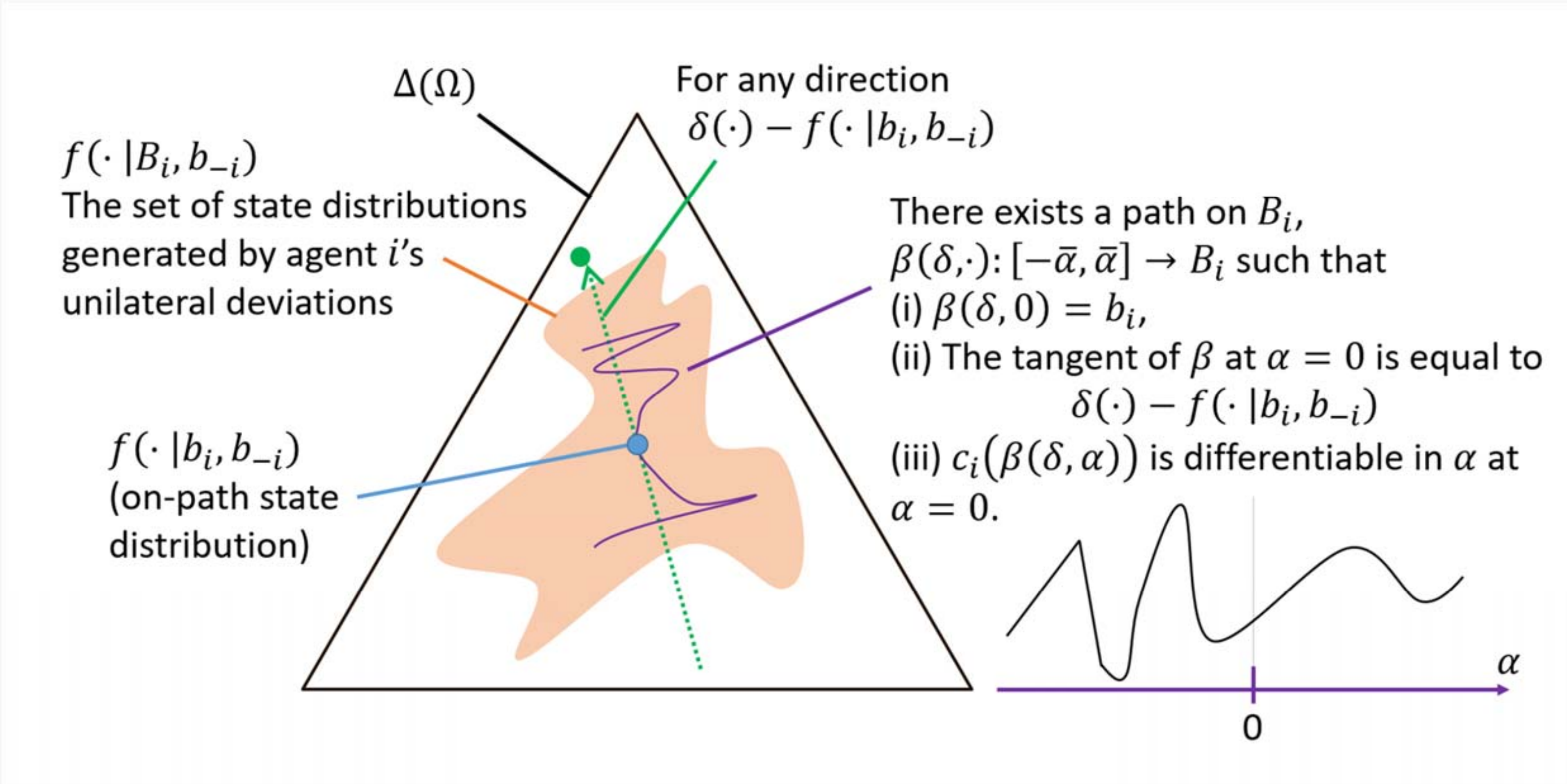
and $c_i(\beta_i(\delta, \alpha))$ is differentiable in α at $\alpha = 0$.

∗Richness (1) sounds very restrictive, but we can replace it with much weaker requirements without loss of substances.


$\Delta(\Omega)$
$f(\cdot\,|B_i, b_{-i})$
The set of state distributions
generated by agent i's
unilateral deviations
$f(\cdot\,|b_i, b_{-i})$
(on-path state
distribution)



$\Delta(\Omega)$
For any direction
$\delta(\cdot) - f(\cdot\,|b_i, b_{-i})$
$f(\cdot\,|B_i, b_{-i})$
The set of state distributions generated by agent i's unilateral deviations
$f(\cdot\,|b_i, b_{-i})$
(on-path state distribution)



$\Delta(\Omega)$
For any direction
$\delta(\cdot) - f(\cdot\,|b_i, b_{-i})$
$f(\cdot\,|B_i, b_{-i})$
The set of state distributions
generated by agent i's
unilateral deviations
There exists a path on B_i,
$\beta(\delta,\cdot): [-\bar{\alpha}, \bar{\alpha}] \rightarrow B_i$ such that
(i) $\beta(\delta, 0) = b_i$,
(ii) The tangent of β at $\alpha = 0$ is equal to
$\delta(\cdot) - f(\cdot\,|b_i, b_{-i})$
(iii) $c_i(\beta(\delta, \alpha))$ is differentiable in α at
$\alpha = 0$.
$f(\cdot\,|b_i, b_{-i})$
(on-path state
distribution)
α
0

Richness: Definition (2)
(Weaker than (1))

Each agent changes state distribution in only finite directions through pure action deviation. However, each agent can change state distribution in all directions through mixed action deviation.

For each $i \in N$, **there exist** $\bar{\alpha} > 0$, $\{\delta^k\}_{k=1}^K$, **and** $\{\beta_i^k : [-\bar{\alpha}, \bar{\alpha}] \to B_i\}_{k=1}^K$ **such that**
$K \equiv \dim(\Delta(\Omega)) = |\Omega| - 1$ **vectors** $\{\delta^k(\cdot) - f(\cdot|b)\}_{k=1}^K$ **are linearly independent,**
for every $k \in \{1,...,K\}$, $\beta_i^k(0) = b_i$,

$$\lim_{\alpha \to 0} \frac{f(\cdot | \beta_i^k(\alpha), b_{-i}) - f(\cdot | b)}{\alpha} = \delta_i^k(\cdot) - f(\cdot | b) \ ,$$

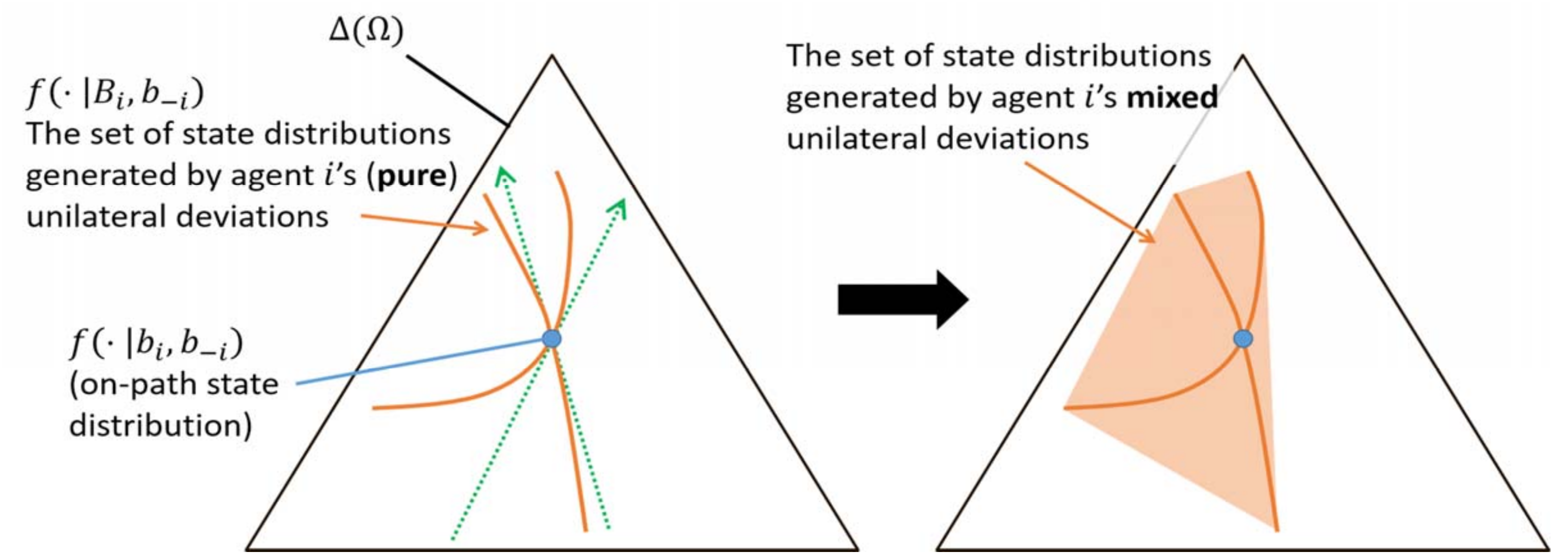
and $c_i(\beta_i^k(\alpha))$ **is differentiable in** $\alpha = 0$.

Alternatively:

An action profile $b \in B$ **is said to be *rich* if for every** $i \in N$ **and** $\delta \in \Delta(\Omega)$, **there exist** $\bar{\alpha} > 0$ **and a path on** B_i , $\beta_i(\delta, \cdot) : [-\bar{\alpha}, \bar{\alpha}] \to \Delta(B_i)$, **such that** $\beta_i(\delta, 0) = b_i$,

(4) $$\lim_{\alpha \to 0} \frac{f(\cdot | \beta_i(\delta, \alpha), b_{-i}) - f(\cdot | b)}{\alpha} = \delta(\cdot) - f(\cdot | b) \ ,$$

and $c_i(\beta_i(\delta, \alpha))$ **is differentiable in** α **at** $\alpha = 0$.



*** We can further replace (2) with a weaker requirements without loss of substances (explain later).**

With richness (1) or (2), we can show:

First-Order Condition:

If (g,x) induces b , for every $i \in N$ and $\delta \in \Delta(\Omega)$,

$$\frac{\partial}{\partial \alpha}\left\{E[v_i(g(\omega),\omega)-x_i(\omega)\,|\,\beta_i(\delta,\alpha),b_{-i}]-c_i(\beta_i(\delta,\alpha))\right\}\Big|_{\alpha=0}=0 \ .$$

Non-Constant Deviation:

For every $i \in N$ and every non-constant function $\xi:\Omega \to R$, there exists $b_i' \neq b_i$ such that

$$E[\xi(\omega)\,|\,b_i',b_{-i}] > E[\xi(\omega)\,|\,b].$$

By adding any non-constant fee, each agent has incentive to deviate.

Ex-post Equivalence Theorem

Theorem 1: Consider an arbitrary $(b,(g,x))$. Suppose that b is rich ((1) or (2)) and (g,x) induces b. For every payment rule $\tilde{x}$, the associated mechanism $(g,\tilde{x})$ induces b if and only if there exists a vector $z \in R^n$ such that for every $\omega \in \Omega$,

$$x(\omega) - \tilde{x}(\omega) = z \quad .$$

Consider an arbitrary (b,g), x, and $\tilde{x}$. Assume both (g,x) and $(g,\tilde{x})$ induce b. Let

$$U_i \equiv E[v_i(g(\omega),\omega) - x_i(\omega) \,|\, b] - c_i(b_i) \text{ and } \tilde{U}_i \equiv E[v_i(g(\omega),\omega) - \tilde{x}_i(\omega) \,|\, b] - c_i(b_i).$$

From Theorem 1, we have:

Ex-post payment equivalence: $\tilde{x}_i(\omega) = x_i(\omega) - U_i + \tilde{U}_i$

Ex-post revenue equivalence: $\sum_{i \in N} \tilde{x}_i(\omega) = \sum_{i \in N} x_i(\omega) - \sum_{i \in N} (U_i - \tilde{U}_i)$

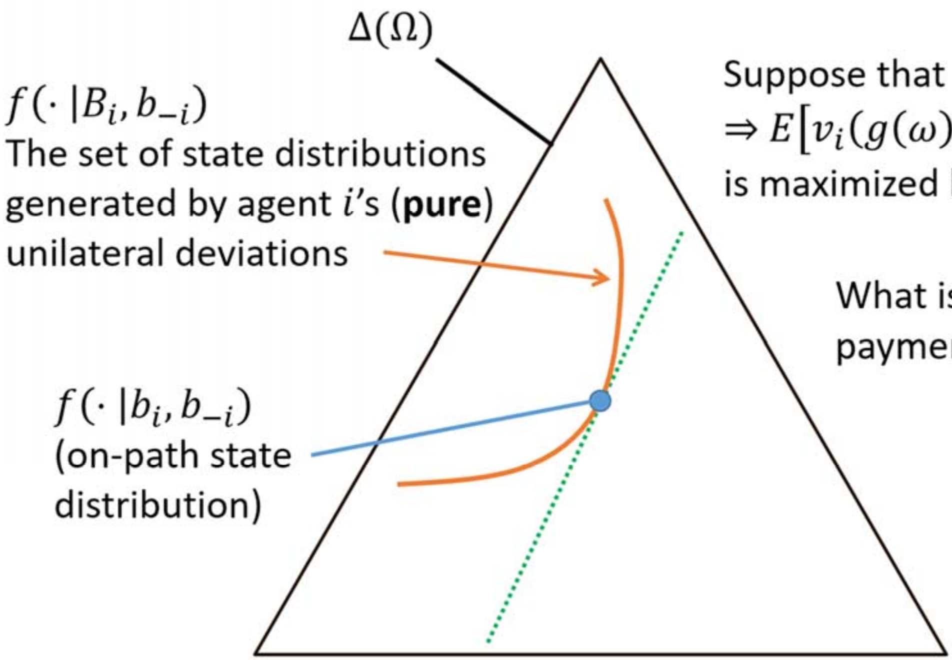
Ex-post payoff equivalence:

$$v_i(g(\omega),\omega) - \tilde{x}_i(\omega) - c_i(b_i)$$
$$= \{v_i(g(\omega),\omega) - x_i(\omega) - c_i(b_i)\} + U_i - \tilde{U}_i$$

Theorem 1 implies that the class of all well-behaved mechanisms is quite limited.

Cf. Green and Laffont (1977, 1979), Holmström (1979): Hidden Information

* Even without richness, inducibility may still be a severe requirement.



Suppose that (g, x) induces b
$\Rightarrow E\left[v_i(g(\omega),\omega) - x_i(\omega)|\tilde{b}_i, b_{-i}\right] - c_i(b_i)$
is maximized by $\tilde{b}_i = b_i$.

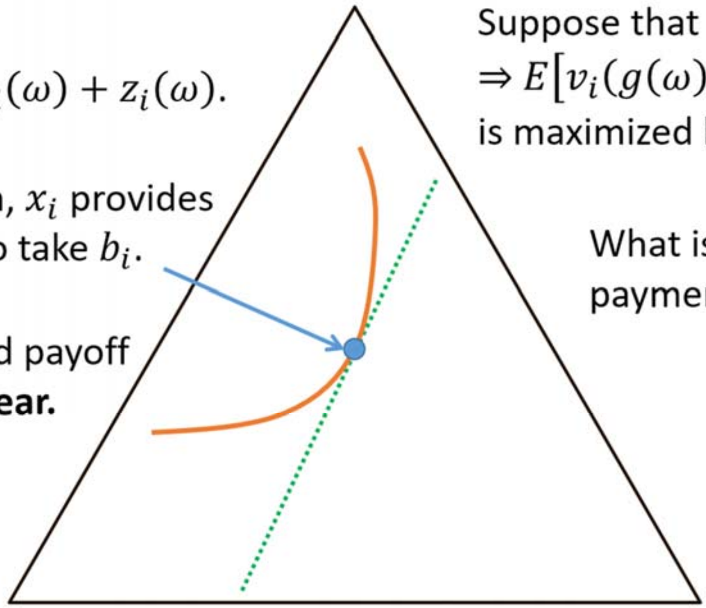
What is the condition for another payment rule $\tilde{x}_i$ to induce b_i?

* Even without richness, inducibility may still be a severe requirement.

Let $\tilde{x}_i(\omega) = x_i(\omega) + z_i(\omega)$.

By assumption, x_i provides an incentive to take b_i.

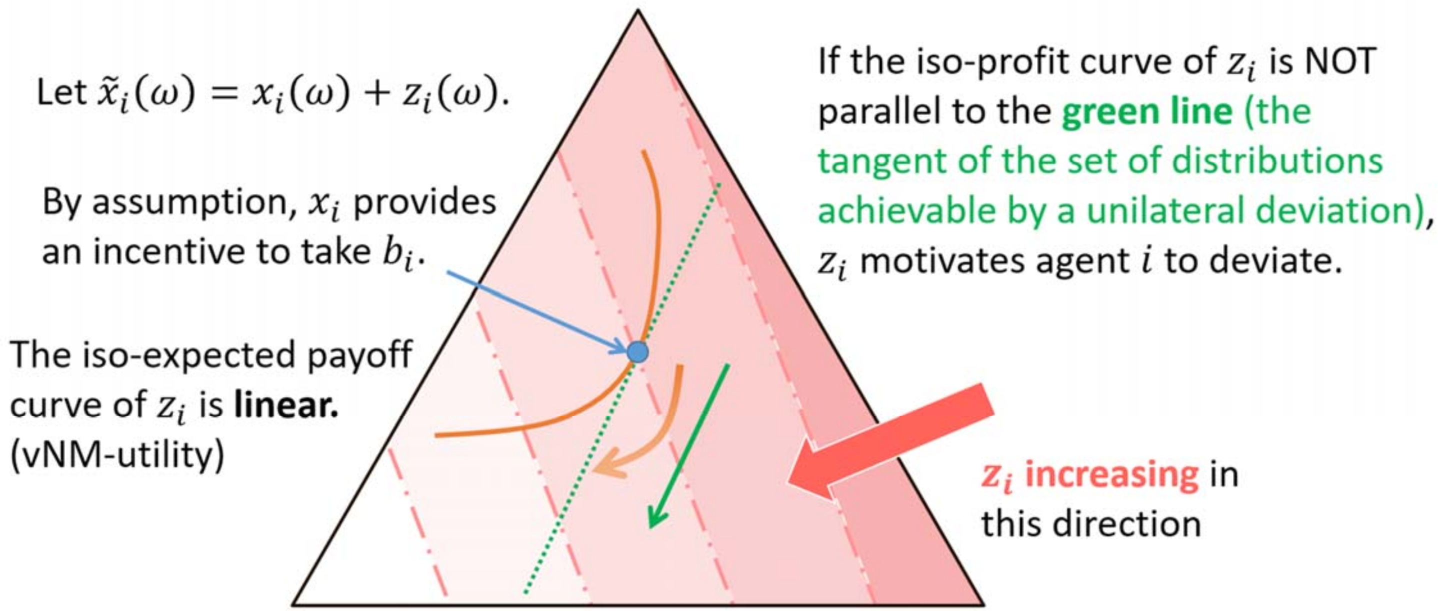
The iso-expected payoff curve of z_i is **linear.** (vNM-utility)



Suppose that (g, x) induces b
$\Rightarrow E\left[v_i(g(\omega), \omega) - x_i(\omega) | \tilde{b}_i, b_{-i}\right] - c_i(b_i)$
is maximized by $\tilde{b}_i = b_i$.

What is the condition for another payment rule $\tilde{x}_i$ to induce b_i?

*** Even without richness,**
inducibility may still be a severe requirement.



*** Even without richness,**
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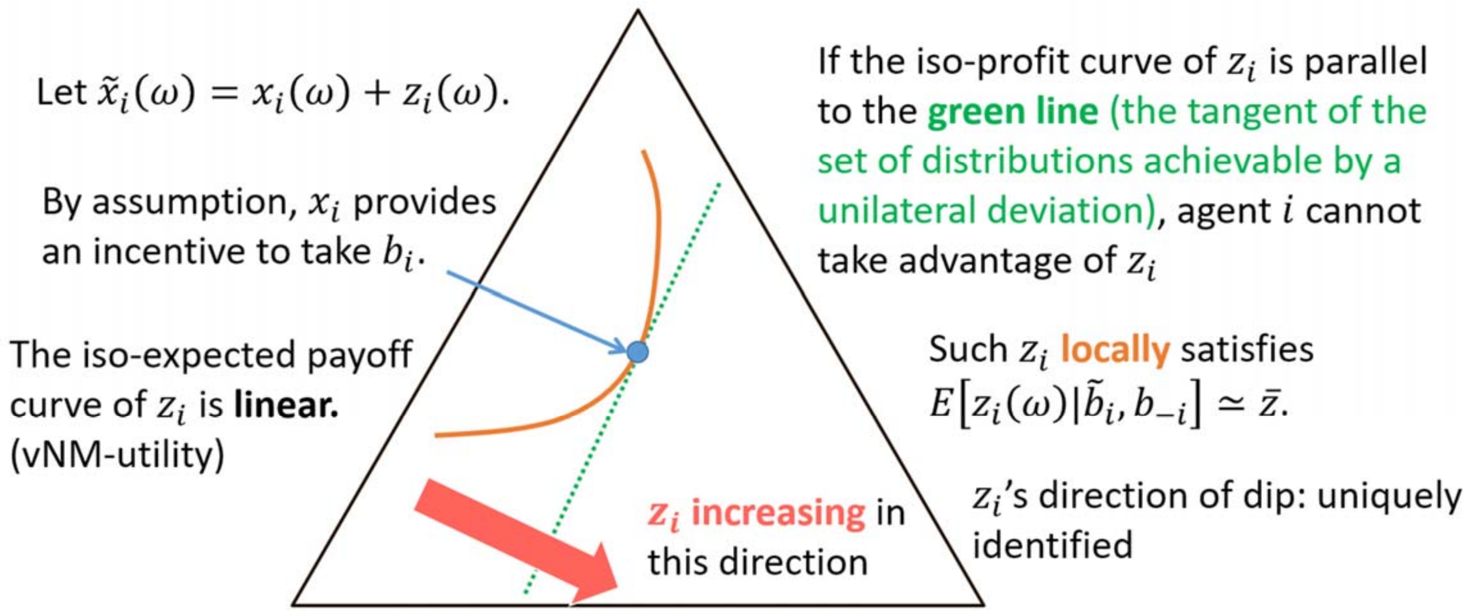
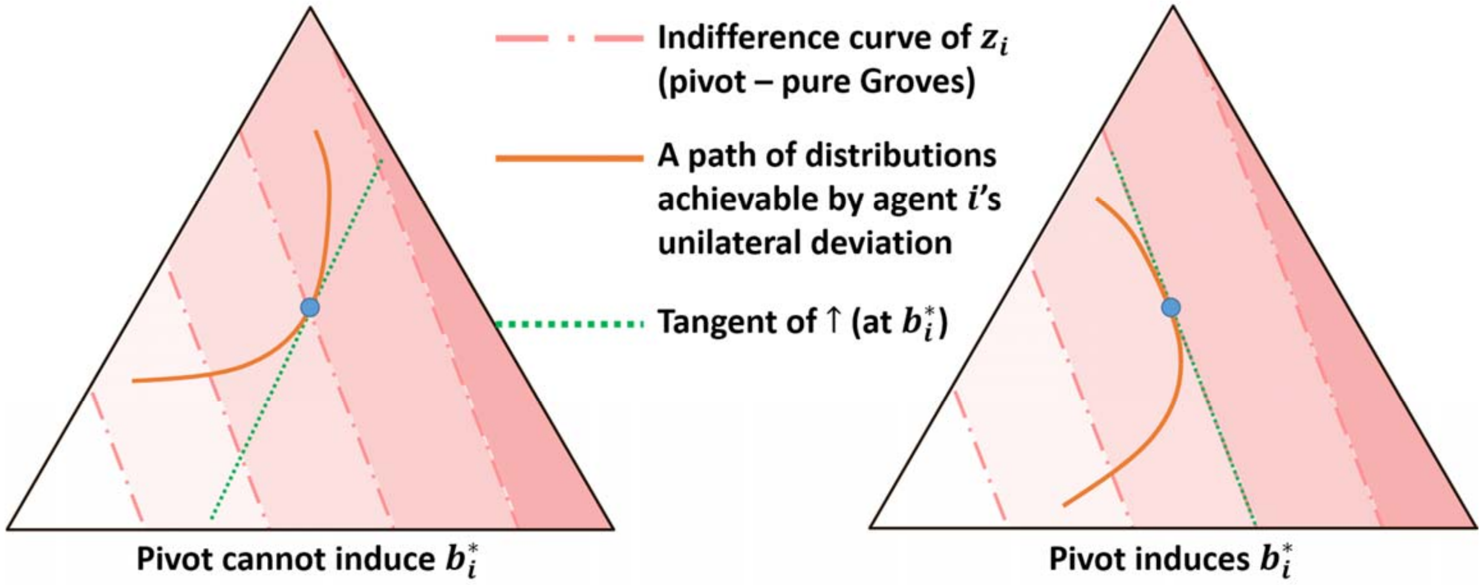


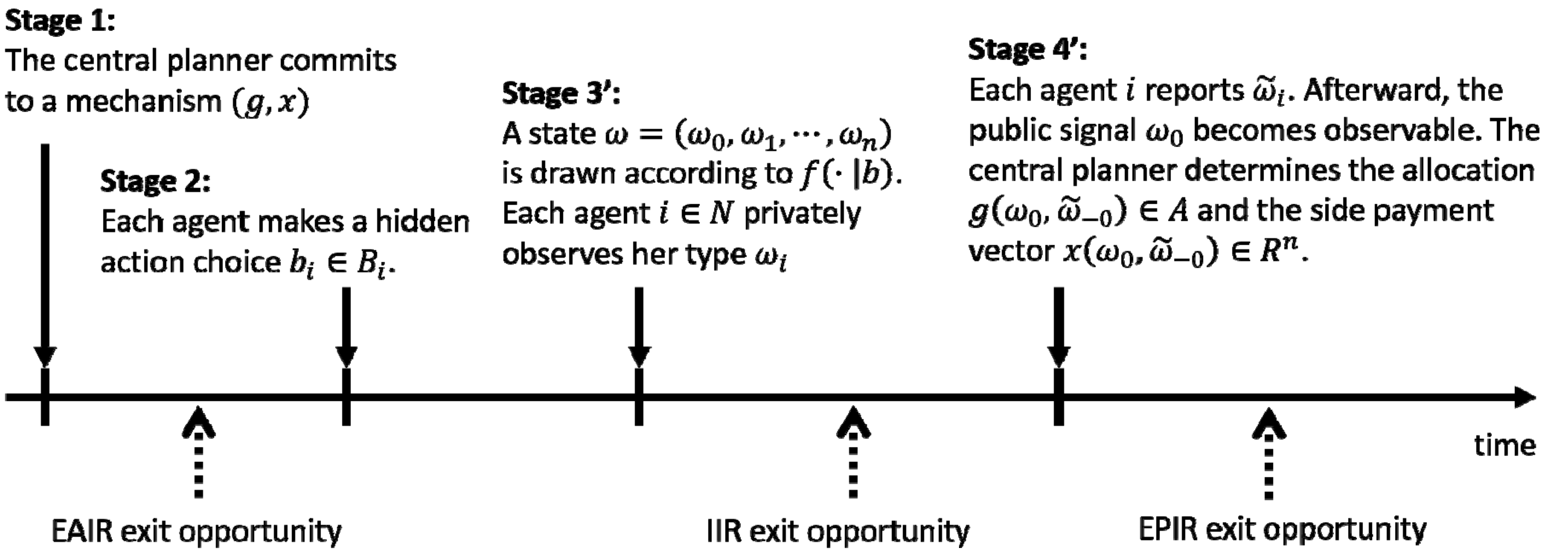
Even without richness, Pivot (or VCG) fails to satisfy inducibility.

$$\text{Let } z_i(\omega) = \min_{a \in A} \sum_{j \in N \cup \{0\}} v_j(a, \omega).$$



4. General Model: Hidden Action and Hidden Information

Timeline (general)



Definition 3 (Ex-Post Incentive Compatibility, EPIC): A mechanism (g,x) is said to be *ex-post incentive compatible (EPIC)* if truth-telling is an ex-post equilibrium; for every $i \in N$, $\omega \in \Omega$, and $\tilde{\omega}_i \in \Omega_i$,

$$v_i(g(\omega),\omega) - x_i(\omega) \geq v_i(g(\tilde{\omega}_i,\omega_{-i}),\omega) - x_i(\tilde{\omega}_i,\omega_{-i}) \ .$$

Definition 4 (Bayesian Implementability, BI): A combination $(b,(g,x))$ is said to be *Bayesian implementable (BI)* if the selection of the action profile b at stage 2 and the truthful revelation at stage 4' results in a Nash equilibrium; for every $i \in N$, every $b_i' \in B_i$, and every function $\sigma_i : \Omega_i \to \Omega_i$,

$$E[v_i(g(\omega),\omega) - x_i(\omega) \,|\, b] - c_i(b_i)$$
$$\geq E[v_i(g(\sigma_i(\omega_i),\omega_{-i}),\omega_{-i}) - x_i(\sigma_i(\omega_i),\omega_{-i}) \,|\, b_i',b_{-i}] - c_i(b_i').$$

Theorem 2: Consider an arbitrary (b,g). Assume b is rich.

1. **Suppose that there exists a payment rule x such that (g,x) induces b and satisfies EPIC. For every payment rule $\tilde{x}$, whenever $(g,\tilde{x})$ induces b, it satisfies EPIC.**
2. **Suppose that there exists a payment rule x such that $(b,(g,x))$ satisfies BI. For every payment rule $\tilde{x}$, whenever $(g,\tilde{x})$ induces b, $(b,(g,\tilde{x}))$ satisfies BI.**

[We find a mechanism that satisfies inducibility but does not satisfy IC]
⇒[We can never find a mechanism that satisfies both]

5. Efficiency: Pure Groves Mechanism

An allocation rule g is said to be *efficient* if

$$\sum_{i\in N\cup\{0\}} v_i(g(\omega),\omega) \geq \sum_{i\in N\cup\{0\}} v_i(a,\omega) \text{ for all } a\in A \text{ and } \omega\in\Omega.$$

A combination (b,g) is said to be *efficient* if g is efficient and the selection of b maximizes the expected welfare:

$$E[\sum_{i\in N\cup\{0\}} v_i(g(\omega),\omega)\,|\,b] - \sum_{i\in N\cup\{0\}} c_i(b_i)$$
$$\geq E[\sum_{i\in N\cup\{0\}} v_i(g(\omega),\omega)\,|\,\tilde{b}] - \sum_{i\in N\cup\{0\}} c_i(\tilde{b}_i) \text{ for all } \tilde{b}\in B.$$

A payment rule x is said to be *Groves* if there exists $y_i:\Omega_{-i}\to R$ for each $i\in N$ such that

$$x_i(\omega) = -\sum_{j\in N\cup\{0\}\setminus\{i\}} v_j(g(\omega),\omega) + y_i(\omega_{-i}).$$

Pure Groves Mechanism: Definition

Groves mechanism with constant fees

A payment rule x is said to be *pure Groves* if there exists a vector $z=(z_i)_{i\in N}\in R^n$ such that

$$x_i(\omega)=-\sum_{j\in N\cup\{0\}\setminus\{i\}} v_j(g(\omega),\omega)+z_i\,.$$

Note Pivot (VCG) mechanism (SPA) is not pure Groves.

In single-unit allocations (with symmetry),
PGM (symmetric) is equivalent to
'Posted Price with Descending Auction determining loser's gain'

With the constraints of efficiency, inducibility, and richness, we can safely focus on pure Groves.

Theorem 3:

Suppose that (b, g) is efficient. For every payment rule x, (g, x) induces b if x is pure Groves.

Suppose that b is rich and (b, g) is efficient. For every payment rule x, (g, x) induces b if and only if x is pure Groves.

(Proof is straightforward from ex-post equivalence.)

With efficiency, inducibility, and richness, it is difficult to satisfy IC (EPIC or BI) in interdependent values, while IC automatically holds in private values.

Theorem 4:

Suppose that b is rich and (b,g) is efficient. There exists a payment rule x such that (g,x) induces b and satisfies EPIC if and only if for every $i \in N$, $\omega \in \Omega$, and $\tilde{\omega}_i \in \Omega_i$,

(8) $$\sum_{j \in N \cup \{0\}} v_j(g(\omega),\omega) \geq v_i(g(\tilde{\omega}_i,\omega_{-i}),\omega) + \sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\tilde{\omega}_i,\omega_{-i}),\tilde{\omega}_i,\omega_{-i}).$$

There exists a payment rule x such that $(b,(g,x))$ satisfies BI if and only if for every $i \in N$, $b_i' \in B_i$, and $\sigma_i : \Omega_i \to \Omega_i$,

(9) $$E[\sum_{j \in N \cup \{0\}} v_j(g(\omega),\omega) \mid b] - c_i(b_i)$$
$$\geq E[v_i(g(\sigma_i(\omega_i),\omega_{-i}),\omega) + \sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\sigma_i(\omega_i),\omega_{-i}),\sigma_i(\omega_i),\omega_{-i}) \mid b_i',b_{-i}] - c_i(b_i')$$

(8) and (9) implies Groves satisfies EPIC.

Theorem 5:

Suppose that (b,g) is efficient. With private values, for every payment rule x, (g,x) induces b and satisfies DIC if x is pure Groves.

Suppose that b is rich and (b,g) is efficient. With private values, for every payment rule x, (g,x) induces b and satisfies DIC if and only if x is pure Groves.

6. Revenues and Deficits: Efficiency in Private Values

CP's ex-post revenue: $v_0(g(\omega),\omega_0)+\sum_{i\in N} x_i(\omega)$

CP's expected revenue: $E[v_0(g(\omega),\omega_0)+\sum_{i\in N} x_i(\omega)\,|\,b]$

Definition 5 (Ex-Ante Individual Rationality): **A combination $(b,(g,x))$ is said to satisfy *ex-ante individual rationality (EAIR)* if**

$$E[v_i(g(\omega),\omega_0,\omega_i)-x_i(\omega)\,|\,b]-c_i(b_i)\geq 0 \text{ for all } i\in N.$$

Definition 6 (Interim Individual Rationality): **A combination $(b,(g,x))$ is said to satisfy *interim individual rationality (IIR)* if**

$$E_{\omega_{-i}}[v_i(g(\omega_i,\omega_{-i}),\omega_0,\omega_i)-x_i(\omega_i,\omega_{-i})\,|\,b,\omega_i]\geq 0 \text{ for all } i\in N \text{ and } \omega_i\in\Omega_i.$$

Definition 7 (Ex-Post Individual Rationality): **A mechanism (g,x) is said to satisfy *ex-post individual rationality (EPIR)* if**

$$v_i(g(\omega),\omega_0,\omega_i)-x_i(\omega)\geq 0 \text{ for all } i\in N \text{ and } \omega\in\Omega.$$

[EPIR] ⇒ [IIR and EAIR]

Proposition 3: Suppose that (b,g) is efficient, b is rich and inducible. With private values, the maximal expected revenues are given by

$$(11)\qquad R^{EPIR} \equiv n \min_{\omega\in\Omega} \sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j) - (n-1)E\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right].$$

$$(12)\qquad R^{IIR} \equiv \sum_{i\in N} \min_{\omega_i\in\Omega_i} E_{\omega_{-i}}\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega_i,\omega_{-i}),\omega_0,\omega_j)\middle| b,\omega_i\right] - (n-1)E\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right].$$

$$(13)\qquad R^{EAIR} \equiv E\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right] - \sum_{j\in N} c_j(b_j).$$

$$(14)\qquad R^{EAIR,IIR} \equiv E\left[v_0(g(\omega),\omega_0)\middle| b\right] + \sum_{i\in N} \min\Bigg\{E\left[v_i(g(\omega),\omega_0,\omega_i)\middle| b\right] - c_i(b_i),$$

$$\min_{\omega_i\in\Omega_i} E_{\omega_{-i}}\left[\sum_{j\in N\cup\{0\}} v_j(g(\omega_i,\omega_{-i}),\omega_0,\omega_j)\middle| b,\omega_i\right] - E\left[\sum_{j\in N\cup\{0\}\setminus\{i\}} v_j(g(\omega),\omega_0,\omega_j)\middle| b\right]\Bigg\}.$$

$$R^{EPIR} \le R^{IIR,EAIR} \le \min[R^{IIR}, R^{EAIR}]$$

With EPIR,
CP fails to achieve efficiency without deficits when a 'null state' exists.

Proposition 4: **Assume private values.**
Suppose that (b,g) is efficient and b is rich. Suppose also $\sum_{i\in N} c_i(b_i) > 0$ and there exists a 'null state' $\underline{\omega} = (\underline{\omega}_0,\dots,\underline{\omega}_n) \in \Omega$ where

$$v_0(a,\underline{\omega}_0) = 0 \text{ for all } a \in A,$$

and for every $i \in N$,

$$v_i(a,\omega_0,\underline{\omega}_i) = 0 \text{ for all } a \in A \text{ and } \omega_0 \in \Omega_0.$$

Then, with EPIR, CP has a deficit in expectation:

$$R^{EPIR} < 0 \quad .$$

cf. Pivot mechanism satisfies EPIR and DIC, but does not satisfy inducibility.

With IIR and EAIR,
maximal expected revenue is irrelevant to inducibility, and
it is generally non-negative.

Proposition 5: **Assume private values and**

Conditional Independence: **For every** $\tilde{b} \in B$ **and** $\omega \in \Omega$,

$$f(\omega \mid \tilde{b}) = \prod_{i \in N \cup \{0\}} f_i(\omega_i \mid \tilde{b}).$$

Suppose that (b, g) **is efficient and** b **is rich. Then,** $R^{IIR,EAIR}$ **is the maximal expected revenue achieved by Groves mechanisms that satisfy IIR and EAIR.**

Proposition 6: **Assume the suppositions in Proposition 4, conditional independence, and the following conditions.**

Non-Negative Valuation: **For every** $i \in N \cup \{0\}$ **and** $\omega \in \Omega$,

$$v_i(g(\omega), \omega_0, \omega_i) \geq 0.$$

Non-Negative Expected Payoff: **For every** $i \in N$,

(15) $$E[v_i(g(\omega), \omega_0, \omega_j) \mid b] \geq c_i(b_i).$$

With IIR and EAIR, the central planner has non-negative expected revenue:

$$R^{IIR,EAIR} \geq 0 \quad .$$

Non-negative valuation excludes the case of bilateral bargaining (Myerson and Satterthwaite (1983)).
Non-negative expected payoff excludes the case of opportunism in hold-up problem (excludes large sunk cost $c_i(b_i)$**).**

7. No Externality: 'Private Richness'

Private Richness implies
each agent i can change the distribution of ω_i (but not ω_{-i}) in all directions.

Independence of information structure: **for every $\omega \in \Omega$ and $b \in B$,**

$$f(\omega \mid b) = f_0(\omega_0)\prod_{i \in N} f_i(\omega_i \mid b_i) \ .$$

Definition 8 (Private Richness): An action profile $b \in B$ is said to be *privately rich* if we have independence of information structure, and for every $i \in N$ and $\delta_i \in \Delta(\Omega_i)$, there exist $\bar{\alpha} > 0$ and $\beta_i(\delta_i,\cdot):[-\bar{\alpha},\bar{\alpha}] \to \Delta(B_i)$ such that $\beta_i(\delta_i,0) = b_i$,

(16) $$\lim_{\alpha \to 0} \frac{f_i(\cdot \mid \beta_i(\delta_i,\alpha)) - f_i(\cdot \mid b_i)}{\alpha} = \delta_i(\cdot) - f_i(\cdot \mid b_i),$$

and $c_i(\beta_i(\delta_i,\alpha))$ is differentiable in α at $\alpha = 0$.

Interim Equivalence Theorem

Proposition 7: **Consider an arbitrary $(b,(g,x))$. Suppose that b is privately rich and (g,x) induces b. For every payment rule $\tilde{x}$, the associated mechanism $(g,\tilde{x})$ induces b if and only if**

$$E_{\omega_{-i}}[x_i(\omega_i,\omega_{-i})-\tilde{x}_i(\omega_i,\omega_{-i})\,|\,b_{-i}] \text{ is independent of } \omega_i.$$

Fix an arbitrary (b,g) and two arbitrary payment rules x and $\tilde{x}$. Assume (g,x) and $(g,\tilde{x})$ induce b. Let

$$U_i \equiv E[v_i(g(\omega),\omega)-x_i(\omega)\,|\,b]-c_i(b_i) \text{ and } \tilde{U}_i \equiv E[v_i(g(\omega),\omega)-\tilde{x}_i(\omega)\,|\,b]-c_i(b_i).$$

We have Interim payment equivalence:

$$E_{\omega_{-i}}[\tilde{x}_i(\omega_i,\omega_{-i})\,|\,b_{-i}] = E_{\omega_{-i}}[x_i(\omega_i,\omega_{-i})\,|\,b_{-i}]-U_i+\tilde{U}_i$$

With private richness, inducibility automatically implies BI.

Proposition 8: **Consider an arbitrary (b,g). Assume private richness. If there exists a payment rule x such that $(b,(g,x))$ satisfies BI, then, for every payment rule $\tilde{x}$, whenever $(g,\tilde{x})$ induces b, $(b,(g,\tilde{x}))$ satisfies BI.**

With efficiency, inducibility, and private richness, we can safely focus on 'Expectation-Groves' instead of PGM.

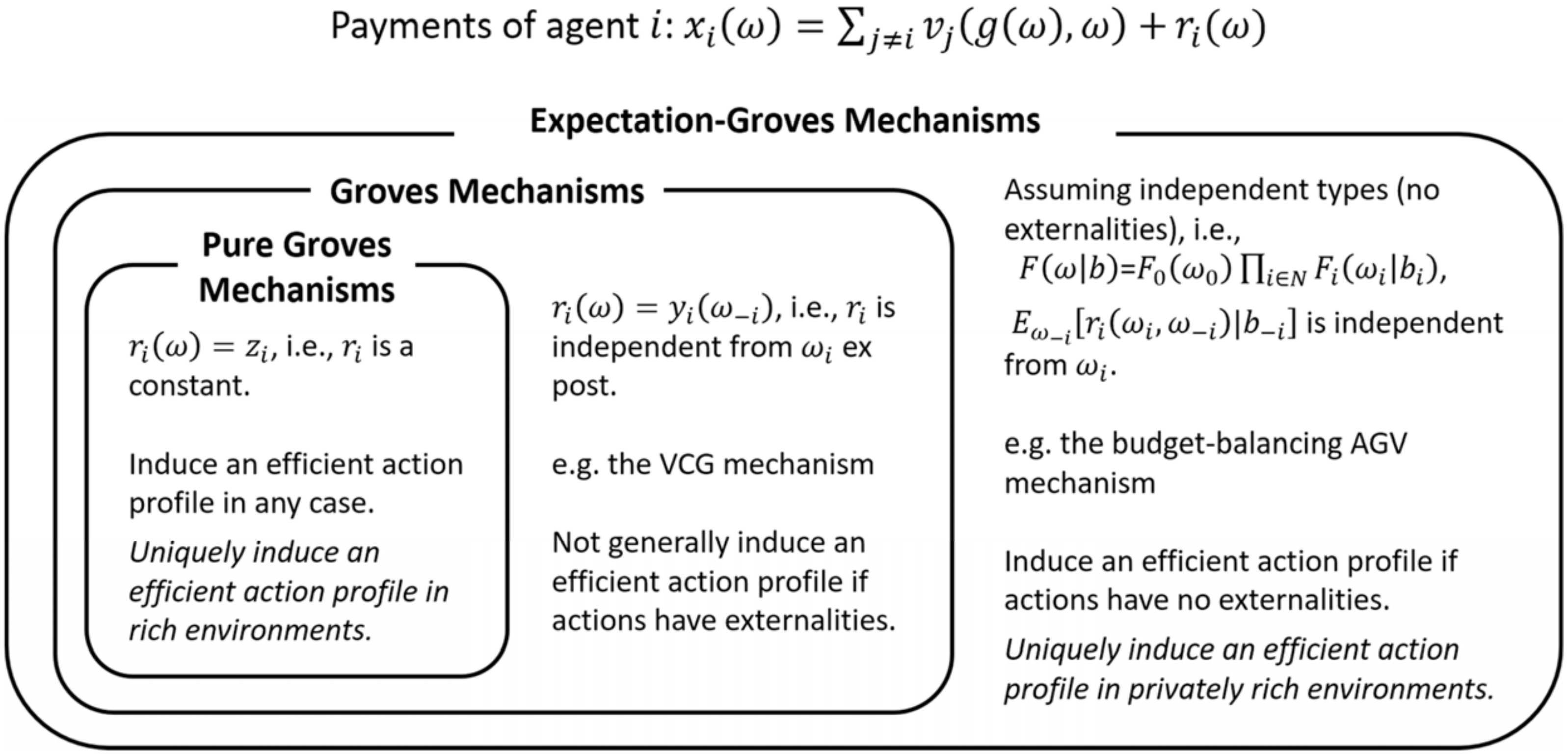
Assume (b,g) is efficient. A payment rule x is said to be *expectation-Groves* if for each $i \in N$, there exist $r_i : \Omega \to R$ such that for every $i \in N$ and $\omega \in \Omega$,

$$x_i(\omega) = -\sum_{j \in N \cup \{0\} \setminus \{i\}} v_j(g(\omega), \omega) + r_i(\omega), \text{ and}$$

$$E_{\omega_{-i}}[r_i(\omega_i, \omega_{-i}) \,|\, b_{-i}] \text{ is independent of } \omega_i.$$

Expectation-Groves guarantees inducibility under private richness.
Groves are expectation-Groves.
Whenever x is expectation-Groves, any payment rule $\tilde{x}$ is expectation-Groves if and only if $E_{\omega_{-i}}[x_i(\omega_i, \omega_{-i}) - \tilde{x}_i(\omega_i, \omega_{-i}) \,|\, b_{-i}]$ is independent of $\omega_i \in \Omega_i$.

Proposition 9:
Suppose that (b,g) is efficient. With independence, (g,x) induces b if x is expectation-Groves.
Suppose that b is privately rich and (b,g) is efficient. With independence, (g,x) induces b if and only if x is expectation-Groves.


Payments of agent i: $x_i(\omega) = \sum_{j\neq i} v_j(g(\omega),\omega) + r_i(\omega)$
Expectation-Groves Mechanisms
Groves Mechanisms
Pure Groves Mechanisms
$r_i(\omega) = z_i$, i.e., r_i is a constant.
Induce an efficient action profile in any case.
Uniquely induce an efficient action profile in rich environments.
$r_i(\omega) = y_i(\omega_{-i})$, i.e., r_i is independent from ω_i ex post.
e.g. the VCG mechanism
Not generally induce an efficient action profile if actions have externalities.
Assuming independent types (no externalities), i.e., $F(\omega|b)$=$F_0(\omega_0)\prod_{i\in N} F_i(\omega_i|b_i)$, $E_{\omega_{-i}}[r_i(\omega_i,\omega_{-i})|b_{-i}]$ is independent from ω_i.
e.g. the budget-balancing AGV mechanism
Induce an efficient action profile if actions have no externalities.
Uniquely induce an efficient action profile in privately rich environments.

With private values,
'Expectation-Groves' guarantees non-negative ex-post payments.

With private values, *AGV mechanism* is expectation-Groves:

(17)
$$r_i(\omega) = \sum_{j\in N\cup\{0\}\setminus\{i\}} v_j(g(\omega),\omega_j) - E_{\tilde{\omega}_{-i}}[\sum_{j\in N\cup\{0\}\setminus\{i\}} v_j(g(\omega_i,\tilde{\omega}_{-i}),\tilde{\omega}_j) \mid b_{-i}]$$
$$+\frac{1}{n-1}\sum_{j\in N\setminus\{i\}} E_{\tilde{\omega}_{-j}}[\sum_{h\in N\cup\{0\}\setminus\{j\}} v_h(g(\omega_j,\tilde{\omega}_{-j}),\tilde{\omega}_h) \mid b_{-j}].$$

AGV satisfies *budget-balancing*:
$$\sum_{i\in N} x_i(\omega) = 0 \quad \text{for all} \quad \omega\in\Omega.$$

With independence,
CP can achieve efficiency
even with the constraints of BI and budget-balancing!

9. Conclusion

We studied mechanism design with side payments that includes hidden action and hidden information. We assumed richness in that each agent has a wide availability of ex-ante activities that have a significant externality effect on other agents' valuations.
We showed that the class of mechanisms that induce the desired action profile is restrictive as follows.

- **The payment rule that satisfies inducibility is unique up to constants. We have the ex-post equivalence theorem.**
- **Efficient mechanisms that satisfy both inducibility and incentive compatibility must be pure Groves, corresponding to a posted-price scheme with descending auction.**
- **It is difficult to satisfy both inducibility and incentive compatibility in interdependent values, while it is generally possible in private values.**
- **CP has to struggle to avoid deficits. But we have possibility results once we permit interim commitments (IIR).**
- **With no externality, expectation-Groves, including AGV, are only well-behaved mechanism.**
- **With no externality and private values, we can achieve efficiency with BI and BB.**

End