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Efficient Combinatorial Allocations: Individual Rationality versus Stability

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Combinatorial Allocation Problem with Incomplete Information

$$(N, A, \Omega),\ \ \Omega \equiv \times_{i \in N} \Omega_i$$

Side Payments are permitted: **Auction**
Multilateral Trading
Incentive Auction
and more …

Both central planner (CP) and participants (players) bring heterogeneous commodities to sell.

Player $i's$ initial endowment e_i

CP's initial endowment e_0

$e_i \cap e_j = \phi$ **for** $i \neq j$

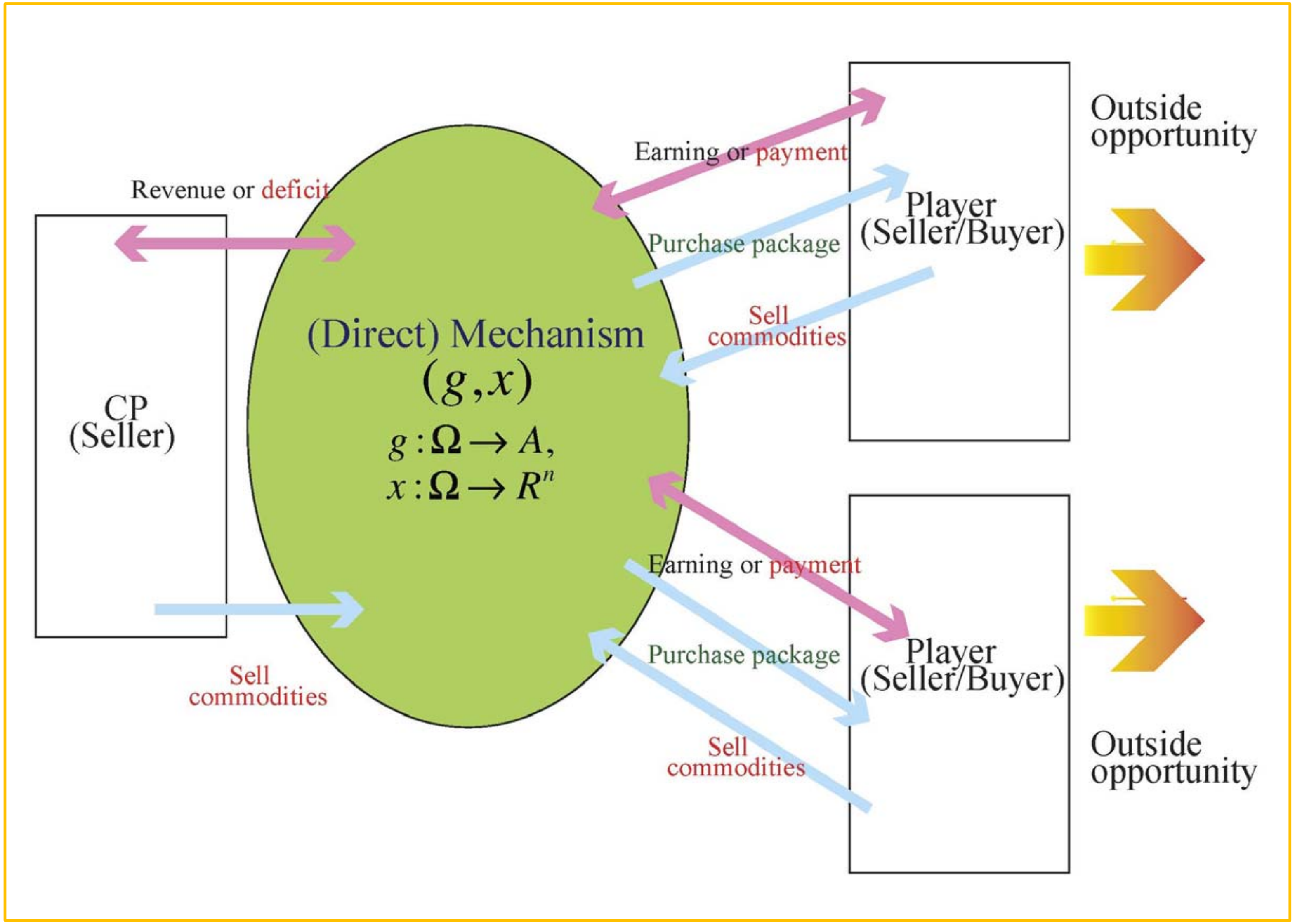
CP has zero valuation for any package of commodities Only participants (players) purchase commodities:

Allocation (package profile) $a = (a_1, ..., a_n) \in A$

$a_i \cap a_j = \phi$

$\bigcup_{i \in N} a_i = \bigcup_{i \in N \cup \{0\}} e_i$

Examples: **Transfer of spectrum licenses from Broadcasting to mobile phone**
Reallocation of old and new airport slots


Outside opportunity
Earning or payment
Revenue or deficit
Player (Seller/Buyer)
Purchase package
Sell commodities
(Direct) Mechanism
(g, x)
$g : \Omega \to A,$
$x : \Omega \to R^n$
CP (Seller)
Earning or payment
Purchase package
Player (Seller/Buyer)
Sell commodities
Sell commodities
Outside opportunity

Basic assumptions:

Quasi-Linearity

Risk-Neutrality for players

Risk-aversion for CP

Private Values and Private goods $v_i(a_i, \omega_i) - t_i$

Independent distribution

Positive Expected Surplus $E[\sum_{i \in N} v_i(f(\omega), \omega_i)] - E[\sum_{i \in N} U_i^*(\omega_i)] > 0$

We assume Payoff-Equivalence: **Williams (1999)**
Krishna and Maenner (2001), et al.

Requirements for a Mechanism (g, x):

Efficiency (E): $$\sum_{i\in N} v_i(g(\omega),\omega_i) = \max_{a\in A} \sum_{i\in N} v_i(a,\omega_i) \text{ for all } \omega.$$

Bayesian Incentive Compatibility (BIC):

$$E[v_i(g(\omega),\omega_i) - x_i(\omega) \mid \omega_i] \geq E[v_i(g(\omega_i',\omega_{-i}),\omega_i) - x_i(\omega_i',\omega_{-i}) \mid \omega_i]$$

for all (i,ω_i,ω_i').

Ex-Ante Individual Rationality (EAIR):

$$E[v_i(g(\omega),\omega_i) - x_i(\omega)] \geq E[v_i(e_i,\omega_i)] \text{ for all } i.$$

Constant Positive Revenue (CPR):

$$\sum_{i\in N} x_i(\omega) > 0 \text{ for all } \omega. \quad \sum_{i\in N} x_i(\omega) \text{ is constant.}$$

∗ With CPR, we can decompose payment rule x into a combination (r, y):

$$\sum_{i \in N} y_i(\omega) = 0 \text{ for all } \omega$$

$$r_i = x_i(\omega) - y_i(\omega) \text{ for all } \omega.$$

$$\sum_{i \in N} r_i(\omega) \text{ is the constant revenue.}$$

This paper investigates, and compares, two distinct decision procedures (1 and 2).

Procedure 1

CP has initiative to design a mechanism.

Players have option to exit from the allocation problem.

Hence, procedure 1 requires a mechanism to satisfy Interim Individual Rationality (IIR):

Interim Individual Rationality (IIR):

$$E[v_i(g(\omega),\omega_i) - x_i(\omega) \,|\, \omega_i] \geq v_i(e_i,\omega_i) \text{ for all } (i,\omega_i)$$

Procedure 2

Players have initiative to design a mechanism collectively.

Players are committed to participate: we do not need to require IIR.

CP sells joint ownership of e_0 for fixed price $\sum_{i \in N} r_i$.

Any (largest) proper coalition can occupy CP's commodities e_0 by excluding the remaining player i at the expense of losing trading opportunity with e_i. (Which is more valuable between e_0 and e_i?)

Hence, procedure 2 requires a mechanism to satisfy a stability condition namely 'Marginal Stability (MS)'.

What is Marginal Stability?

For every coalition $S \subset N$, we define $A(S) \subset A$ as

$$[a \in A(S)] \Leftrightarrow [a_{N \setminus S} = e_{N \setminus S}].$$

We define the value of coalition S when it occupies e_0 at the expense of $e_{N \setminus S}$ by

$$\varpi(S) \equiv E[\max_{a \in A(S)} \sum_{i \in S} v_i(a_i, \omega_i)].$$

Marginal Stability (MS):

$$E[\sum_{j \in N \setminus \{i\}} \{v_j(g_j(\omega), \omega_j) - y_j(\omega)\}] \geq \varpi(N \setminus \{i\}) \text{ for all } i.$$

Strict Stability:

$$E[\sum_{j \in S} \{v_j(g_j(\omega), \omega_j) - y_j(\omega)\}] \geq \varpi(S) \text{ for all } S \subset N.$$

∗ When commodities are substitutes, MS implies strict stability.

Purpose of This Paper

We clarify a necessary and sufficient condition for procedure 1 to achieve efficiency.

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We then compare these conditions.

Opt-Out-Type Assumption (Makowski and Ostroy (89), Segal and Whinston (2012)):

Each player i has opt-out type $\omega_i^* \in \Omega_i$:

$$g_i(\omega_i^*, \omega_{-i}) = e_i \text{ for all } \omega_{-i} \in \Omega_{-i}.$$

Main Theorem

There exists an efficient mechanism in procedure 1 (BIC, CPR, and IIR)
if and only if

$$(n-1)\omega(N) < \sum_{i \in N} \varpi(N \setminus \{i\}).$$

There exists an efficient mechanism in procedure 2 (BIC, EAIR, CPR, and MS)
if and only if

$$(n-1)\omega(N) \geq \sum_{i \in N} \varpi(N \setminus \{i\}).$$

Hence,

Procedure 1 can achieve efficiency if and only if procedure 2 cannot.

Sketch of Proof

Procedure 1: From payoff-equivalence, Groves mechanisms, and presence of opt-out types, the maximal revenue is given by

$$-(n-1)E[\sum_{i\in N} v_i(g(\omega),\omega_i)] - \sum_{i\in N} \max_{\omega_i\in\Omega_i}\{v_i(e_i,\omega_i) - E[\sum_{j\in N} v_j(g(\omega),\omega_j)\,|\,\omega_i]\}$$

$$= -(n-1)\omega(N) + \sum_{i\in N} \varpi(N\setminus\{i\}).$$

Procedure 2: MS is equivalent to:

$$-(n-1)\omega(N) + \sum_{i\in N} \varpi(N\setminus\{i\}) \le 0.$$

Implication

$(n-1)\omega(N)$ is greater than $\sum_{i\in N}\varpi(N\setminus\{i\})$ (i.e., procedure 1 is better than 2)

if and almost only if

CP's commodities e_0 are valuable compared with any player's commodities e_i.

Procedure 2 is unsuitable for Auction:

Since any player brings nothing, any (largest) proper coalition is willing to occupy e_0.

Procedure 2 is suitable for multilateral trading:

Since CP brings nothing, any (largest) proper coalition dislikes to lose the trading opportunity with any player.

Main theorem shows general characterization for which is the better procedure:

$$(n-1)\omega(N) \gtreqless \sum_{i\in N}\varpi(N\setminus\{i\})$$