Incentives in Hedge Funds

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**Hedge Fund as Delegated Portfolio Management**

Investor (Unsophisticated)  1 Unit of Fund, No Withdrawal

Manager  

- M Units of Personal Fund: Manage Investor’s and Personal Funds
- ‘Separate Management’ or ‘Equity Stake’
- Weak Regulation, Low Transparency
- Generate Alpha

Manager

\[
\begin{align*}
\text{Skilled Type} & : \text{Select Alpha (Action) } a \in [0, \infty) \text{ with Non-Pecuniary Cost } C(a) \\
\text{Unskilled Type} & : \text{Alpha 0}
\end{align*}
\]
Incentive Problem

Hidden Type: Investor Cannot Identify whether Manager is skilled or not
 Hidden Activity: Investor Cannot Observe Manager’s Activity

Q: Can We Solve Incentive Problem?

A: Yes, but We Need Capital Gain Tax!

HF Survives:
Positive Capital Gain Tax (Fulcrum Scheme or Equity Stake)

Skilled Entry
Unskilled Exit
Skilled and Investor
Pareto Improvement
No HF
No CG Tax
Investor Entry
Unskilled

A: Yes, but We Need Capital Gain Tax!
Manager’s Incentive Fee Scheme

\[ y : [0, \infty) \to [-M, \infty], \quad y(x) \in [-M, \infty) \]

Return-Contingency, Penalty, Escrow for Solvency
Real Fee Scheme

‘2:20’ Scheme
Asymmetry, No Penalty, Convexity, High-Powered

\[ y(x) = 0.2x + 0.02 \]

Criticisms (Warren Buffet): ‘2:20’ Makes Manager More Risk-Taking by Side Contracting with Third Party. We Should Change ‘2:20’ Scheme to

‘Fulcrum’ Scheme
Symmetric, Positive Penalty, Linear, Low-Powered

\[ y(x) = k(x - 1) \]
Side Contracting: Performance Mimicry

Randomize Return

Cumulative Distribution \( F : [0, \infty) \rightarrow [0, 1] \)

\[ E[z \mid F] = x \]
**Example (Lo (2001))**

**Capital Decimation Partners (CDP)**

Unskilled Can Generate Alpha \( \frac{p}{1-p} > 0 \) with Prob. \( 1 - p \)

**Diagram:**

- **Unskilled Manager**
  - Safe Asset 1 Unit (HF)
  - Safe Asset \( p \) Unit
  - Safe Asset \( p^2 \) Unit

- **Escrow**
- **Arbitrageur**

**Covered Option Sale:**
Transfer Safe Asset to Arbitrageur if S&P500 Index Decline 20% (Prob. \( p \))

- Price \( p \)
- Price \( p^2 \)
- Price \( p^3 \)
Previous Works: Hedge Fund Never Survives

Foster + Young (08/09)  With No CG Tax, No Scheme Can Solve Incentive Problem

Media:  FT (18/3/08), NYT (3/8/08)

“HF Never Survives. We Need More Transparency!”
Results of This Paper

- CG Tax Functions
  - With No CG Tax, We Cannot Solve Incentive Problem (a la Foster + Young)
  - With Positive CGT Rate $t > 0$, We Can Solve Incentive Problem

- Constrained Optimal Scheme
  - Fulcrum After Taxation: Low-Powered

- Income Tax on Fee Functions
  - Income Tax Rate Should be Greater than CG Tax Rate, $\tau > t$
  - Manager Selects Constrained Optimal Scheme Voluntarily

- Equity Stake Functions
  - We Can Solve Incentive Problem without Fulcrum
**Assumption: Separate Management**

**Skilled Manager**

- **HF Return**: $a + 1$
- **Personal Fund**: $M - w(y)$
- **Escrow**: $w(y)$
- **Action**: $a$
- **Cost**: $c(a)$
- **Side Contract**: $F$
- **Random Return**: $z$
- **Return**: $(M - w(y))(a' + 1)$

**Unskilled Manager**

- **HF Return**: $1$
- **Personal Fund**: $M - w(y)$
- **Escrow**: $w(y)$
- **Action**: $a = 0$
- **Action**: $a' = 0$
- **Side Contract**: $F$
- **Random Return**: $z$
- **Return**: $w(y)$
- **Alpha**: $0$
Incentive Problem: Five Constraints

① Skilled Entry
② Unskilled Exit
③ Investor Entry
④ Welfare Improvement
⑤ Skilled Non-mimicry: Skilled Needs No Third-Party Side Contract
**Skilled Entry:** $V(y, t, \tau) \geq \overline{V}(t)$

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**Outside Opportunity**
Manage Entire Personal Fund $M$

Payoff
$\overline{V}(t) \equiv M\{(1-t)\overline{a}(1-t) - c(\overline{a}(1-t))\}$

CG Tax $tM\overline{a}(1-t)$

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**HF Industry**
Put $w(y)$ in Escrow, Unmanageable

Payoff
$V(y, \tau, t) \equiv \min[(1-\tau)\overline{y}(a^*(y, \tau) + 1), \overline{y}(a^*(y, \tau) + 1) - c(a^*(y, \tau))]$

$+\{M - w(y)\}\{(1-t)\overline{a}(1-t) - c(\overline{a}(1-t))\}$

CG Tax $t\{M - w(y)\}\overline{a}(1-t)$

Income Tax $\max[\tau, \overline{y}(a^*(y, \tau) + 1), 0]$

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$\overline{a}(1-t)$ Maximize $(1-t)a - c(a)$

$a^*(y, \tau)$ Maximize $(1-\tau)y(a + 1) - c(a)$
Unskilled Exit: \( \max_{F \in \Phi} E[\min[(1 - \tau)y(z), y(z)] | F] \leq 0 \)

HF Industry
No Skill but Side Contracting

Payoff
\( \max_{F \in \Phi} E[\min[(1 - \tau)y(z), y(z)] | F] \)

CG Tax 0
Income Tax \( E[\max[\tau y(z), 0]] | F] \)
**Investor Entry:** $U(y,t,\tau) \geq 0$, i.e., $a^*(y,\tau) \geq y(a^*(y,\tau) + 1)$

**HF Industry**

**Payoff**

$U(y,t,\tau) \equiv \min\{(1-t)\{a^*(y,\tau) - y(a^*(y,\tau) + 1)\},
\quad a^*(y,\tau) - y(a^*(y,\tau) + 1)\}$

**CG Tax** $\max\{t\{a^*(y,\tau) - y(a^*(y,\tau) + 1)\}, 0\}$
Welfare Improvement: $S(y,t,\tau) > \bar{S}$

No HF (Status Quo)

\[ t = \tau = 0 \]

Surplus
\[ \bar{S} \equiv M\{\bar{a}(1) - c(\bar{a}(1))\} \]

Surplus Increases

HF Industry

Surplus
\[ S(y,t,\tau) \equiv a^*(y,\tau) - c(a^*(y,\tau)) + \{M - w(y)\}\{\bar{a}(1) - t) - c(\bar{a}(1 - t))\} \]
**No Capital Gain Tax: Impossibility**

**Theorem:** Suppose CGT Rate $t = 0$. Then, There Exists No Fee Scheme that Satisfies Skilled Entry, Unskilled Exit, and Welfare Improvement.

**Outline of Proof:** Assume $a > 0$ is only available, $y(0) = -w(y)$

### Skilled
- Alpha $a$
- $y(a + 1) - C(a)$
- $-w(y)(a - c(a))$

### Skilled Entry
- $y(a + 1) \geq w(y)a + \{1 - w(y)\}c(a)$
- $> w(y)a$
- **Contradiction!**

### Unskilled (CDP)
- Alpha $a$ Pro. $\frac{1}{a + 1}$
- $y(a + 1)$

### Unskilled Exit
- $\frac{1}{a + 1}y(a + 1) \leq \frac{a}{a + 1}w(y)$

### Skilled
- Alpha $a$ Pro. $\frac{a}{a + 1}$
- $y(0)$

- Put $w(y)$ in Escrow
- $-w(y)(a - c(a))$

- Skilled Entry
- $y(a + 1) \geq w(y)a + \{1 - w(y)\}c(a)$
- $> w(y)a$
- **Contradiction!**
Positive Capital Gain Tax: Possibility

Theorem: There exist Tax Rates \((t, \tau) \in [0,1]^2\) and Fee Scheme \(y \in Y^*(\tau)\) that satisfy All Constraints.

Outline of Proof: Assume \(a > 0\) is only available

- Skilled’s Outside Opportunity
  - Manage Entire Personal Fund
  - Pay CG Tax \(tMa\)

- Skilled’s HF
  - Put \(w(y)\) in Escrow
  - Pay CG Tax \(t\{M - w(y)\}a\)

- Save CG Tax \(tw(y)a\)

“Larger Fund + Less Active” is Better Than “Smaller Fund + More Active”
**Constrained Optimization:** \((y^*, t^*, \tau^*)\)

(1) **Fulcrum Scheme after Taxation**

\[
y(x) = x - 1 \quad \text{for all} \quad x \in [1, \infty)
\]

\[
y(x) = (1 - \tau)(x - 1) \quad \text{for all} \quad x \in [0, 1)
\]

(2) **Skilled Entry Binding**

\[
V(y, t, \tau) = \bar{V}(t)
\]

We Specify \((y, t, \tau) = (y^*, t^*, \tau^*)\) As Maximizing Surplus \(S(y, t, \tau)\) Subject to (1) and (2)

**Theorem:** \((y^*, t^*, \tau^*)\) Satisfies All Constraints. There exists No \((y, t, \tau)\) that Satisfies All Constraints and \(S(y, t, \tau) > S(y^*, t^*, \tau^*)\).
Constrained Optimization: Properties

- Manager is Willing to Select $y^*$ Voluntarily: $y^*$ is the Only Scheme that Satisfies Skilled Entry, Unskilled Exit, Investor Entry, and Skilled Non-mimicry.

- Manager Prefers to Put Personal Fund in Escrow as Large as Possible, Distorting Welfare.

- Income Tax Rate $\tau^*$ is Greater than CG Tax Rate $t^*$: High Income Tax Rate
Another Assumption: Equity Stake

Skilled Manager

Unskilled Manager
**We Don’t Need Penalty, But CG Tax and Big Stake**

**Theorem:** Suppose CGT Rate $t = 0$. Then, There Exists No Fee Scheme that Satisfies Skilled Entry, Unskilled Exit, and Welfare Improvement.

Additional Assumption: $a > 0$ is only available, $\tau = 0$

**Theorem:** For Sufficiently Large Personal Fund $M$, There exist $(t, y)$ that Levy No Penalty but Satisfy All Constraints.

**Outline of Proof:** CDP Must be Covered by Not only Investor’s Fund But also Personal Fund

<table>
<thead>
<tr>
<th>Equity Stake $M$ (Sufficiently Large)</th>
<th>Unskilled’s CDP Covered by Equity Stake</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return $M(a + 1)$</td>
<td>CG Tax $tMa$</td>
</tr>
<tr>
<td>(Prob. $\frac{1}{a+1}$)</td>
<td>(Prob. $\frac{a}{a+1}$)</td>
</tr>
</tbody>
</table>

Expected Return $M - tMa < M$
## Further Comments

**Investor’s Optimization**

- Investor Prefers **higher**-Powered and **More** Penalty than Constrained Optimal Scheme.

- By Transferring Total Tax Revenue to Investor, Government Can Incentivize Investor to Select Constrained Optimal Scheme Voluntarily.

- Investor’s Payoff May be **Greater** than Manager’s Payoff per Unit: Manager May Fold HF Business.

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**Entry Cost**

Entry Cost Functions, if, and Only if, It is **Non-Pecuniary**!