Auctions with Ethical Concerns

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Motivation

1) Implementation of Social Choice Rules

The central planner is unaware of the state, whereas multiple agents obtain their respective partial information.
The central planner asks them to reveal their information.

Without side payments:

Impossibility Theorems (Gibbard, 1973; Satterthwaite, 1975)

Only dictatorship is implementable in dominant strategy

With side payments:

The central planner designs a side payment rule to incentivize agents to be honest.

Possibility Theorems:

Private values: **Vickrey (1961), Clarke (19719, Groves (1973)**

VCG mechanisms
Efficiency (total surplus maximization) is implementable in dominant strategy.

Interdependent Values: **Cremer and McLean (1985, 1988)**

Generalized VCG mechanism
In single-item auctions with single-crossing property, efficiency is implementable in ex-post equilibrium.

2) Ethical Social Choice Rules

The central planner considers social welfare ethically, rather than financially. Consumer sovereignty and monetary equivalent should be limited. Redistribution is prohibited.

Ex. Allocation of ventilators in pandemics
Achievement of low environmental Loads

3) Conflicting Ethical Criteria

Multiple ethical criteria coexist, which are conflicting with one another.

Ex. Triage and reserve in emergency (Pathak et al., 2020)

The central planner has to predetermine an SCR as a reasonable compromise between conflicting ethical criteria.

The Purpose of this Study

Can we implement ethical SCRs?

Side payment rule design plays an important role in implementation of ethical SCRs.

Multiunit auction	**Single-unit demand**
	Private values for WTP
	Interdependent values for welfare evaluation

With single-crossing property, any reasonable SCR is implementable in dominant strategy.

We define a reasonable SCR as a rule that can be derived from "The method of procedure" (Matsushima, 2021)

The central planner makes a compromise between criteria according to the following steps.

- **The central planner predetermines a priority order over criteria, i.e., a procedure.**
- **According to the procedure, the central planner sequentially assigns the commodity to the top-ranked agent at the corresponding criterion.**
- **The central planner determines the value of the SCR as the set of all agents assigned to the commodities through these steps.**

**From the viewpoint of inter-problem regularities,
Matsushima (2021) axiomatically showed that
The method of procedure is dominant for determining an SCR
when multiple conflicting ethical criteria coexist.**

Difficulty of Ethical Implementation

An SCR induced by the method of procedure cannot be considered as the result of maximizing a single objective function.

This study demonstrates a new side-payment rule design:

- **Virtually, for each agent, we prepare as many markets as there are criteria.**
- **For each market, we endogenously set a quantity of commodity to be sold.**
- **We set the price for each market as a variant of the uniform price.**
- **Each agent can purchase a single unit in any market.**
- **Importantly, the agent only has to pay the lowest price among all markets, irrespective of which market they purchase, i.e., which criterion justifies their purchases.**

General Problems

$N = \{1, ..., n\}$	**Set of agents**	$n \geq 2$
A	**Set of allocations**	$a \in A$

We fix a specific allocation problem (A, N).

Ω_i	**Set of types for agent**	$i \in N$
$\Omega = \Omega_1 \times \cdots \times \Omega_n$	**Set of states**	
	$\omega = (\omega_i)_{i \in N} \in \Omega$	

The central planner is unaware of the state, whereas each agent knows their own types.

$C : \Omega \to A$ **Social choice rule (SCR)**

$C(\omega) \in A$ **implies the desirable allocation at state** ω**.**

$t = (t_i)_{i \in N}$ **Side-payment rule**

$t_i : \Omega \to R$

(C, t) **Direct mechanism**

Each agent $i \in N$ **announces a message** $\tilde{\omega}_i \in \Omega_i$ **. The central planner then determines the allocation** $C(\tilde{\omega}) \in A$ **and makes the monetary payment** $t_i(\tilde{\omega}) \in R$ **to each agent** $i \in N$**.**

$v_i(a,\omega_i)+s_i$ **Agent $i's$ quasi-linear utility with private values $(a,s_i)\in A\times R$**

The direct mechanism (C,t) is said to be strategy-proof if truth-telling is a dominant strategy, i.e., for every $i\in N$, $\omega\in\Omega$, and $\omega_i'\in\Omega_i$,

$$v_i(C(\omega),\omega_i)+t_i(\omega)\geq v_i(C(\omega_i',\omega_{-i}),\omega_i)+t_i(\omega_i',\omega_{-i}).$$

An SCR C is said to be implementable in dominant strategy if there exists a side-payment rule t such that (C,t) is strategy-proof.

Assumption 1 (Single Ethical Criterion):

There exists a state-dependent vector of welfare weights $(w_i(\omega))_{i\in N} \in R_+^n$ where $C(\omega) \in A$ is determined as

$$\sum_{i\in N} w_i(\omega)v_i(C(\omega),\omega_i) \geq \sum_{i\in N} w_i(\omega)v_i(a,\omega_i) \text{ for all } a \in A.$$

- **The central planner's concern is in-kind achievement.**
- **The central planner forbids redistribution.**
- $w_i(\omega)v_i(a,\omega_i)$ **is not (necessarily) the monetary equivalent.**
- **The central planner does not consider transfers in welfare evaluation.**

Condition 1 (State-Independence):

The ethical criterion $(w_i(\omega))_{i\in N,\omega\in\Omega}$ is independent of the state $\omega\in\Omega$:

There exists a vector $(w_i)_{i\in N}\in R_+^n$ such that

$$w_i = w_i(\omega) \quad \text{for all } i\in N \text{ and } \omega\in\Omega.$$

Theorem 1:

With single ethical criterion (Assumption 1) and state-independence (Condition 1), the direct mechanism (C,t) is strategy-proof if for every $i\in N$, there exists $e_i:\Omega_{-i}\to R$ such that

$$t_i(\omega)=\sum_{j\neq i}\frac{w_j}{w_i}v_j(C(\omega),\omega_j)+e_i(\omega_{-i}) \quad \text{for all } \omega\in\Omega.$$

Intuition:

- **Owing to state-independence, we can extend the internalization by the VCG mechanism from homogeneity to heterogeneity in welfare weight.**

Single-Unit Auction

Single-unit auction

$$A = N$$
$$\Omega_i = [0,1]$$
$$v_i(i,\omega_i) = \omega_i$$
$$v_i(j,\omega_i) = 0 \quad \text{for} \quad j \neq i\,.$$

We have:

$$C(\omega) = i \qquad \text{if} \quad w_i(\omega)\omega_i > w_j(\omega)\omega_j \quad \text{for all} \quad j \in N \setminus \{i\}\,.$$

Condition 2 (Single-Crossing Property):

For each $i \in N$, $\omega_{-i} \in \Omega_{-i}$, and $j \in N \setminus \{i\}$,

$$w_i(\omega)\omega_i - w_j(\omega)\omega_j \text{ is increasing in } \omega_i.$$

Single-crossing property is weaker than Condition 1:
It permits state-dependent welfare weights.

Single-crossing property holds both for WTP and for welfare evaluation.

We define the threshold $\omega_i(\omega_{-i}) \in \Omega_i$ by

$$w_i(\omega_i(\omega_{-i}), \omega_{-i})\omega_i(\omega_{-i}) = \max_{j \neq i}[w_j(\omega_i(\omega_{-i}), \omega_{-i})\omega_j].$$

Owing to single-crossing property, $\omega_i(\omega_{-i}) \in \Omega_i$ exists uniquely.

We have:

$$C(\omega) = i \qquad \text{if } \omega_i > \omega_i(\omega_{-i}).$$

The central planner specifies a side-payment rule t by

$$t_i(\omega) = -\frac{\max_{j \neq i}[w_j(\omega_i(\omega_{-i}), \omega_{-i})\omega_j]}{w_i(\omega_i(\omega_{-i}), \omega_{-i})} \qquad \text{if } C(\omega) = i$$

$$t_i(\omega) = 0 \qquad \text{if } C(\omega) \neq i$$

Theorem 2:

Under single ethical criterion (Assumption 1) and single-crossing property (Condition 2), the specified direct mechanism (C, t) is strategy-proof.

Intuition:

- **The value $\dfrac{\max_{j\neq i}[w_j(\omega_i(\omega_{-i}),\omega_{-i})\omega_j]}{w_i(\omega_i(\omega_{-i}),\omega_{-i})}$ is independent of ω_i. This along with the definition of $\omega_i(\omega_{-i})$ is crucial for the proof.**
- **The mechanism is a natural extension of the second-price auction from homogeneity to heterogeneity in welfare weight.**
- **The mechanism is related to the generalized VCG mechanism (Cremer and McLean, 1985, 1988), which assumes**

  interdependent values for WTP
  interdependent values for welfare evaluation
  homogeneity in welfare weight
  ex-post equilibrium.
- **This study assumes private values for WTP**

  interdependent values for welfare evaluation
  heterogeneity in welfare weight
  dominant strategy.

Multiunit Auction

Multiunit auction $H\ (< n)$ **units of homogeneous commodity**

Single-unit demand

$$A = \{a \subset N \mid |a| = H\}$$

$$\Omega_i = [0,1]$$

$$v_i(a, \omega_i) = \omega_i \qquad \text{if } \ i \in a$$

$$v_i(a, \omega_i) = 0 \qquad \text{if } \ i \notin a$$

We denote h^{th} **greatest welfare evaluation except agent** i **by**

$$\max_{j \neq i}[w_j(\omega)\omega_j \mid h].$$

We have:

$$i \in C(\omega) \qquad \text{if } \ w_i(\omega)\omega_i > \max_{j \neq i}[w_j(\omega)\omega_j \mid H]$$

$$i \notin C(\omega) \qquad \text{if } \ w_i(\omega)\omega_i < \max_{j \neq i}[w_j(\omega)\omega_j \mid H]$$

We define the threshold $\omega_i(h,\omega_{-i}) \in \Omega_i$ **by**

$$w_i(\omega_i(h,\omega_{-i}),\omega_{-i})\omega_i(h,\omega_{-i})$$
$$= \max_{j \neq i}[w_j(\omega_i(h,\omega_{-i}),\omega_{-i})\omega_j \mid h]$$

Owing to single-crossing property, $\omega_i(h,\omega_{-i}) \in \Omega_i$ **exists uniquely.**

We have:

$$i \in C(\omega) \qquad \text{if } \omega_i > \omega_i(H,\omega_{-i})$$
$$i \notin C(\omega) \qquad \text{if } \omega_i < \omega_i(H,\omega_{-i})$$

The central planner specifies a side-payment rule t by

$$t_i(\omega) = -\frac{\max\limits_{j \neq i}[w_j(\omega_i(H,\omega_{-i}),\omega_{-i})\omega_j \mid H]}{w_i(\omega_i(H,\omega_{-i}),\omega_{-i})} \quad \text{if } i \in C(\omega)$$

$$t_i(\omega) = 0 \quad \text{if } i \notin C(\omega)$$

Theorem 3:

Under single ethical criterion (Assumption 1) and single-crossing property (Condition 2), the specified direct mechanism (C, t) is strategy-proof.

Intuition:

- The value $\dfrac{\max_{j \neq i}[w_j(\omega_i(H,\omega_{-i}),\omega_{-i})\omega_j \mid H]}{w_i(\omega_i(H,\omega_{-i}),\omega_{-i})}$ is independent of ω_i. This along with the definition of $\omega_i(H,\omega_{-i})$ is crucial for the proof.
- The mechanism is a natural extension of the uniform-price auction from homogeneity to heterogeneity in welfare weight.
- The mechanism is a natural extension of the generalized VCG mechanism (Theorem 2) from single-unit auction to multiunit auction.

Multiple Criteria

Multiunit auction with a single-unit demand

Assumption 2 (Multiple Ethical Criteria):
There exist $\bar{d}$ ($\leq H$) different ethical criteria:
For each criterion $d \in D \equiv \{1,...,\bar{d}\}$, we have a state-dependent vector of welfare weights $(w_i^d(\omega))_{i \in N} \in R_+^n$.

Associated with each criterion $d \in D$, we define the priority order over agents $\pi_d(\omega): N \to \{1,...,n\}$ as follows:
For each $i \in N$ and $j \in N \setminus \{i\}$,

$$\pi_d(i,\omega) < \pi_d(j,\omega) \qquad \text{if} \quad w_i^d(\omega)\omega_i > w_j^d(\omega)\omega_j .$$

To specify an SCR, the central planner adapts
the method of procedure (Matsushima, 2021).

$\gamma : \{1, ..., H\} \rightarrow D$ **Procedure (priority order over criteria)**

- **In step 1, the top-ranked agent at criterion** $\gamma(1) \in D$ **is selected. This agent is denoted by** $\tau(1, \omega) \in N$ **.**
- **At each step** $h \in \{2, ..., n\}$**, the top-ranked agent at criterion** $\gamma(h) \in D$ **among remaining agents is selected. This agent is denoted by** $\tau(h, \omega)$**.**
- **We specify**

$$C(\omega) = \{\tau(1, \omega), ..., \tau(H, \omega)\} \in N$$

The central planner specifies t_i according to the following steps.

- **In step 1, the top-ranked agent at criterion $\gamma(1)\in D$ among $N\setminus\{i\}$ is selected. This agent is denoted by $\tau(1,\omega_{-i},i)\in N\setminus\{i\}$.**
- **At each step $h\in\{2,...,n-1\}$, the top-ranked agent at criterion $\gamma(h)\in D$ among remaining agents is selected. This agent is denoted by $\tau(h,\omega_{-i},i)$.**
- **We select $\tau(i,d,\omega_{-i})\in N\setminus\{i\}$ and $h(i,d,\omega_{-i})\in\{1,...,H\}$ in the manner that**

$$\tau(i,d,\omega_{-i})=\tau(h(i,d,\omega_{-i}),\omega_{-i},i),$$
$$\gamma(h(i,d,\omega_{-i}))=d,$$
$$\gamma(h)\neq d \text{ for all } h\in\{h(i,d,\omega_{-i})+1,...,H\}.$$

- **In other words, the agent $\tau(i,d,\omega_{-i})$ is selected in step $h(i,d,\omega_{-i})$, justified by criterion d. After step $h(i,d,\omega_{-i})$, criterion d is never used to justify the assignment. Hence, we can regard agent $\tau(i,d,\omega_{-i})$ as the last agent to be assigned and justified by criterion d, provided agent i is absent.**

- **We denote the priority of the agent $\tau(i,d,\omega_{-i})$ at criterion d among all agents except for agent i by**

$$H(i,d,\omega_{-i}) \in \{1,...,H\}.$$

- **We have:**

$i \in C(\omega)$ **if**

$$\omega_i > \min_{d \in D} \frac{\max_{j \neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})}$$

$i \notin C(\omega)$ **if**

$$\omega_i < \min_{d \in D} \frac{\max_{j \neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})}$$

The central planner specifies a side-payment rule t as follows:

$$t_i(\omega) = -\min_{d\in D} \frac{\max_{j\neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})} \quad \text{if } i \in C(\omega)$$

$$t_i(\omega) = 0 \quad \text{if } i \notin C(\omega)$$

Interpretation:

Virtually, for each agent i, we have $\bar{d}$ markets, at which $H(i,d,\omega_{-i})$ units are sold for the price given by

$$\frac{\max_{j\neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})}.$$

Importantly, agent i can purchase the commodity for the lowest price across markets, irrespective of in which market to purchase, i.e., which criterion to justify.

Theorem 4:

With multiple ethical criteria (Assumption 2) and single crossing property (Condition 2), the specified direct mechanism (C,t) is strategy-proof.

Intuition:

- **The value** $\min_{d\in D}\frac{\max_{j\neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})}$ **is independent of** ω_i**. This, along with the definitions of** $\omega_i(H(i,d,\omega_{-i}),\omega_{-i})$**, is crucial for the proof.**

Subsidies and Set-Asides

Consideration of more criteria will change who become winners, and how much winners have to pay, in state-dependent manner, through two routes: subsidies and set-asides (Ayres and Cramton, 1996; Pai and Vohra, 2012; Athey et al., 2013)

Example 1 (Subsidies): $D = \{1,2\}$, $\omega_i = 1$

$$w_i^1 = n - i + 1$$

$$w_i^2 = n - i + 1 + y \qquad \text{if } i \in \{1,\dots,H\}$$

$$w_i^2 = n - i + 1 \qquad \text{if } i \in \{H+1,\dots,n\}.$$

Criteria 1 and 2 have no conflict in assignment.

The assignment to agents $i = 1,2,\dots,H$ is optimal.

Criterion 2 makes more subsidies to them:

$$t_i(\omega) = -\frac{n-H}{n-i+1+y} > -\frac{n-H}{n-i+1}$$

Example 2 (Set-asides):

Replace criterion 2 with

$$w_i^2 = i \quad \text{for each} \quad i \in \{1, ..., H\}$$

Criteria 1 and 2 are in great conflict.
Use criterion 2 only in step H.
Agent n is justified by criterion 2 (Set-asides).
Agent $i \in \{1, 2, ..., H-1\}$ is justified by criterion 1.
Criterion 2 increases their prices:

$$t_i(\omega) = -\frac{w_H^1}{w_i^1} = -\frac{n-H+1}{n-i+1} < -\frac{n-H}{n-i+1}$$