Assignments with Ethical Concerns

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Motivation

COVID-19 Pandemic

Triage for Assigning Ventilators in US (Pathak et al, 2020)

On-site decision prioritized medical personnel over the existing triage.

Some states introduced a reserve system as an alternative to the triage.

Scarce Resources in Emergency, Global Environment, Life, Dignity ...

(Ventilators, Vaccines, Recycled materials, Public Housing, Educations …)

How do we allocate such scarce resources as welfare-optimum?

What does social welfare mean?

How do we compromise multiple ethical criteria?

Willingness to Pay as an Ethical Criterion

High WTPs are prioritized to assign scarce resources.

WTP is a popular and convenient criterion for economists.

Objections to WTP

WTP alone cannot represent social welfare.

WTP becomes the ethical dictator.

Various other criteria: Medical personnel,
Underlying illness,
Seniority, Child …

All have their own characteristics, but they may conflict with one another.

Question

How does the central planner make a compromise

between the conflicting criteria

and link it to a persuasive social decision?

Background (Broad Sense)

Skepticism toward Consumer Sovereignty, Monetary Equivalent, Selfishness, and Rationality

Impartial observers (Smith, 1749/1969), Ethical preferences (Harsanyi, 1955), Merit goods (Musgrave, 1957, 1987), Primary goods (Vickrey, 1960; Rawls, 1971, 1988), Community preferences (Colm, 1965), Specific egalitarianism (Tobin, 1970), Random allocations (Weitzman, 1977), Commitment (Sen, 1977), Libertarian paternalism (Thaler and Sunstein, 2003, 2008), Social common capital (Uzawa, 2008) ……

Growing Concerns

Global warming, Triage, Affirmative action, Refugee resettlement ……

Approach

In-Kind Assignment Problem with a Single-Unit Demand

$N = \{1, \ldots, n\}$	**Set of agents.** $n \geq 3$
$I \subset N$	**Set of participants**
q	**Number of slots.** $q \leq \lvert I \rvert$
(I, q)	**Assignment Problem**
X	**Set of all assignment problems**

$C : X \to 2^N$

Social choice rule (SCR)

$C(I, q) \subset I$

$|C(I,q)| = q$.

Agent in $C(I,q)$ obtains a slot.

We take an axiomatic approach to characterize SCRs that respect the ethical criteria.

Basic Axioms on SCR

Axiom 1:

The same agents obtain slots when the number of slots increases:

For every $(I,q) \in X$ and $i \in I$,

$$[i \in C(I,q)] \Rightarrow [i \in C(I,q+1)].$$

Axiom 2:

The same agents obtain slots when the set of participants becomes smaller:

For every $(I,q) \in X$, $i \in I$, and $j \in I \setminus \{i\}$,

$$[i \in C(I,q)] \Rightarrow [i \in C(I \setminus \{j\},q)].$$

We focus on SCRs that satisfy Axioms 1 and 2.

Basic Characterization

Behind any SCR, we can envision a procedure

for prioritizing some (artificially created) criteria.

$D = \{1, 2, ..., \bar{d}\}$ **Set of criteria**

$\pi_d : \{1, ..., n\} \to N$ **Priority order over agents at criterion** $d \in D$

Agent $\pi_d(h) \in N$ **has** h^{th} **rank for** $d \in D$.

$\pi = (\pi_d)_{d \in D}$

$\gamma : \{1, 2, ..., n\} \to D$ **Procedure (i.e., priority order over criteria)**

$\Gamma = (D, \pi, \gamma)$

We define SCR $C = C^{\Gamma}$ according to the following steps:

- **Consider an arbitrary problem $(I, q) \in X$.**
- **In step 1, the top-ranked agent at criterion $\gamma(1) \in D$ is selected. This agent is denoted by $i(1)$.**
- **In each step $k \in \{2, ..., q\}$, the top-ranked agent at criterion $\gamma(k) \in D$ among remaining participants is selected. This agent is denoted by $i(k)$.**
- **We define SCR C^{Γ} by $C^{\Gamma}(I, q) = \{i(1), ..., i(q)\}$.**

Theorem 1:

An SCR C satisfies Axioms 1 and 2 if and only if there exists $\Gamma = (D, \pi, \gamma)$ such that $C = C^{\Gamma}$.

Intuition:

- **Any SCR with Axioms 1 and 2 can be induced by a common priority order over criteria γ for various problems, by making up multiple criteria and their priority orders over agents (D, π) appropriately, and artificially.**

Example:

Assume $n = 3$. Consider SCR C given by

	I={1,2,3}	I={1,2}	I={1,3}	I={2,3}
q=1	{3}	{1}	{3}	{3}
q=2	{2,3}			

Clearly, C satisfies Axioms 1 and 2.

However, it lacks transitivity (see the red cases.)

Specify $D = \{1, 2\}$ and π as

π_1	3	1	2
π_2	3	2	1

By letting $\gamma(1) = 1$ and $\gamma(2) = 2$, we have $C = C^{\Gamma}$ (i.e., rationalize C).

Contributions

We shall fix (D,π) as the existing criteria.

Denote C^{γ} instead of C^{Γ}, where $\Gamma=(D,\pi,\gamma)$.

We axiomatize SCRs C by requiring additional inter-problem axioms regarding how C to reflect the existing criteria (D,π).

This study introduces two distinct methods to induce reasonable SCRs:

Method of Procedure and Method of Aggregation

Method of Aggregation:

Generalization of triage

We quantify the criteria so that they can be compared.

We aggregate these quantities.

Method of Procedure:

Generalization of reserve system

We select a procedure as a priority order over criteria.

According to the procedure, we sequentially pick up a single agent, who is top-ranked for the corresponding criterion.

Main Results (Selected)

- **The method of aggregation emphasizes consistent respect for individual agents across different problems.**
- **The method of procedure emphasizes consistent respect for individual criteria (not agents) across different problems.**
- **These methods are incompatible with one another. Only ethical dictatorships are induced from both methods.**
- **The method of aggregation is superior when we can utilize detailed information about welfare evaluations such as comparability.**
- **The method of procedure is superior when there are informational limitations.**

Method of Procedure

$\delta = (\delta(I,q))_{(I,q)\in X}$

Justification

$\delta(I,q): C(I,q) \rightarrow D$

The assignment to agent $i \in C(I,q)$ is justified by criterion $\delta(I,q)(i) \in D$.

Axiom 3 (Fair Justification):

There exists a justification δ that satisfies three properties:

Respecting Priority, Diversity, and Invariance

Property (i) (Respecting Priority):

Why an agent $i \in C(I,q)$ was assigned can be explained by their priorities implied by criterion $\delta(I,q)(i)$:

For every $(I,q) \in X$, $i \in C(I,q)$, and $j \in I \setminus C(I,q)$,

$$\pi^{-1}_{\delta(I,q)(i)}(i) < \pi^{-1}_{\delta(I,q)(i)}(j).$$

Property (ii) (Diversity):

For each criterion $d \in D$, the number of assigned agents who are justified by d is unaffected by which agents participate:

For every $(I, q) \in X$ and $d \in D$,

$$\left|\left\{ i \in C(I,q) \mid \delta(I,q)(i) = d \right\}\right| = \left|\left\{ i \in C(N,q) \mid \delta(N,q)(i) = d \right\}\right| .$$

Property (iii) (Invariance):

The criterion $\delta(I,q)(i)$ that justifies an assigned agent $i \in C(I,q)$ is unchanged as q increases:

For every $(I,q) \in X$ and $i \in C(I,q)$,

$$\delta(I,q)(i) = \delta(I,q+1)(i).$$

Theorem 2:

An SCR C satisfies Axioms 1, 2, and 3 if and only if a procedure γ exists such that $C = C^{\gamma}$.

Intuition:

- **From property (i), each criterion justifies the assignments in order from the highest rank.**
- **From properties (ii) and (iii), which agents to be assigned ($C(I,q) \subset I$) is determined by a common priority order over criteria ($\gamma : \{1,...,n\} \to N$) for different problems ($(I,q) \in X$).**
- **Owing to these properties, an SCR C can be induced by some procedure γ associated with the pre-existing criteria (D,π).**

Method of Aggregation

Axiom 4 (Agent Consistency):

The same agents can obtain slots when the set of participants becomes larger and the same number of slots as this increase are added:

For every $(I, q) \in X$, $i \in I$, and $j \in I \setminus \{i\}$,

$$[\, i \in C(I, q) \,] \Rightarrow [\, i \in C(I \cup \{j\}, q+1) \,].$$

$f:\{1,2,...,n\}\rightarrow N$ **Aggregation (i.e., priority order over agents)**

$C = C^f$ **SCR induced by aggregation** f

$$C(I,q)=\{i\in I \mid f^{-1}(i)\leq q\}$$

Higher-rank agent for f **has higher priority.**

Theorem 3:

An SCR C satisfies Axioms 1, 2 and 4 if and only if there exists an aggregation f such that $C = C^f$.

Intuition:

- **Axiom 4 requires an SCR be induced by a common priority order over agents ($f : \{1, 2, ..., n\} \rightarrow N$) for different problems.**
- **Any SCR that satisfies Axioms 1, 2, and 4 must be in the form of C^f.**

Ethical Dictatorship

SCR C is said to be ethical-dictatorial for criterion $d \in D$ if for every $(I, q) \in X$,

$$C(I, q) = \{i \in I \mid \pi_d^{-1}(i) \leq q\}.$$

SCR C is said to be ethical-dictatorial if there exists $d \in D$ such that it is ethical-dictatorial for d.

Theorem 4:

An SCR C satisfies Axioms 1, 2, 3, and 4 if and only if it is ethical-dictatorial.

Intuition:

- **Axiom 3 requires that a procedure (i.e., priority order over criteria) γ exists.**
- **Axiom 4 requires that an aggregation (i.e., priority order over agents) f exists.**
- **For an SCR to satisfy both axioms, the aggregation f and the first-step criterion (i.e., priority order over agents) $\gamma(1)$ must be equivalent:**
  For every $k \in \{2, ..., n\}$, if $f(k') \notin I$ for all $k' \in \{1, ..., k-1\}$ and $f(k) \in I$, then $\pi_{\gamma(1)}(k) = f(k)$ must hold.

From these arguments, we obtained the following results:

- **The method of procedure emphasizes consistent respect for individual criteria across problems.**
- **The method of aggregation emphasizes consistent respect for individual agents across problems.**
- **Applying both methods together, it is inevitable to prioritize a single criterion and neglect other criteria (ethical dictatorship).**

State-Dependent Social Choice Rule (SSCR)

Ω	**Set of states**
$\omega = (\omega_d)_{d \in D} \in \Omega$	$\omega_d : N \to R$
$\omega_d(i) \in R$	**Evaluation of agent** $i \in N$ **at criterion** d.
$\pi = \pi(\omega)$	$\omega_d(\pi_d(h)) > \omega_d(\pi_d(h+1))$

$C = C(\omega) = C(\cdot;\omega)$ **State-dependent social choice rule (SSCR)**

$C(I,q) = C(I,q;\omega)$

$\gamma = \gamma(\omega)$, $\Gamma = \Gamma(\omega) = (D,\pi(\omega),\gamma(\omega))$,

$f = f(\omega)$, $C^{\gamma} = C^{\gamma(\omega)}$, $C^{f} = C^{f(\omega)}$ ……

Axiom 5 (Independence):

The assignment choice does not depend on the evaluation of non-participants:

For every $\omega \in \Omega$, $\omega' \in \Omega$, **and** $(I,q) \in X$,

$$[\omega(i) = \omega'(i) \text{ for all } i \in I] \Rightarrow [C(I,q;\omega) = C(I,q;\omega')].$$

Axiom 6 (Comparability):

If an agent is evaluated outstandingly high by a particular criterion, they are assigned:

For every $\omega \in \Omega$, $d \in D$, and $i \in N$, if

$$\pi_d(\omega_d)(1) = i\,,$$

then a sufficiently large $l > 0$ exists such that for every $\omega' \in \Omega$,

$$[\,\omega'_d(i) \geq \omega_d(i) + l\,,$$

$$\omega'_d(j) = \omega_d(j) \quad \text{for all} \quad j \neq i\,, \text{ and}$$

$$\omega'_{d'} = \omega_{d'} \quad \text{for all} \quad d' \in D \setminus \{d\}\,]$$

$$\Rightarrow [\,C(N,1;\omega') = \{i\}\,].$$

The method of procedure is inconsistent with Axioms 5 and 6.

Theorem 5:

There exists no SSCR C that satisfies Axioms 1, 2, 3, 5, and 6.

Intuition:

- **According to Axiom 6, if a criterion suggests that an agent should be given an exceptional priority, the central planner must disregard whether this criterion has a low priority and give them priority over anyone else.**
- **This contradicts property (ii) in Axiom 3, which, for any problem, and for any criterion, requires a certain number of assigned agents who are justified by this criterion.**

Example 3: **Consider $n = 3$, $\bar{d} = 2$, ω, and ω' given by**

	1	2	3
ω_1	1000	10	5
ω_2	0	5	10
ω'_1	0	10	5
ω'_2	1000	5	10

Consider an arbitrary procedure γ. Since 1000 is sufficiently large, from Axiom 6, we have

$$C^{\gamma(\omega)}(\{1,2,3\},1;\omega) = \{1\}, \quad \gamma(\omega)(1) = 1,$$

$$C^{\gamma(\omega')}(\{1,2,3\},1;\omega') = \{1\}, \text{ and } \gamma(\omega')(1) = 2.$$

Hence, we have

$$C^{\gamma(\omega)}(\{2,3\},1;\omega) = \{2\} \text{ and } C^{\gamma(\omega')}(\{2,3\},1;\omega') = \{3\}.$$

This contradicts Axiom 5, because $\omega_1(2) = \omega'_1(2)$ and $\omega_1(3) = \omega'_1(3)$.

The method of aggregation is compatible with Axioms 5 and 6.

Example (Triage):

$m : R^{\bar{d}} \rightarrow R$ priority point system

Evaluation is aggregated into $m(\omega(i)) \in R$.

$f = f^{m}$ State-dependent aggregation

$[\, f^{m}(\omega)(i) < f^{m}(\omega)(j)\,] \Leftrightarrow [\, m(\omega(i)) > m(\omega(j))\,]$

Clearly, $C^{f^{m}}$ satisfies Axioms 5 and 6.

Informational Basis

Axiom 7 (Ethical Pareto):

If agent i has better rank than agent j at all criteria, agent i has precedence over agent j:

For every $\omega \in \Omega$, $(I,q) \in X$, $i \in I$, and $j \in I \setminus \{i\}$,

$$[\, i \in C(I,q;\omega) \text{ and } \pi^{-1}(\omega)(i) > \pi^{-1}(\omega)(j) \,]$$

$$\Rightarrow [\, j \in C(I,q;\omega) \,], \text{ where } \pi^{-1}(\omega)(i) \equiv (\pi_d^{-1}(\omega)(i))_{d \in D}$$

Axiom 8 (Ordinality without Comparability):

An SSCR $C(\omega)$ depends on the state ω only through its ordinal aspect $\pi(\omega)$:

For every $\omega \in \Omega$ and $\omega' \in \Omega$,

$$[\pi(\omega) = \pi(\omega')] \Rightarrow [C(\omega) = C(\omega')]$$

Theorem 6:

An SSCR C satisfies Axioms 2, 4, 7, and 8 if and only if there exists $d \in D$ for which $C(\omega)$ is ethical-dictatorial in all states.

Intuition: **We can apply Arrow's impossibility theorem (1951).**

From these arguments, we obtained the following results:

- **The method of aggregation is superior to the method of procedure when we can utilize detailed information such as comparability**
- **The method of procedure is superior to the method of aggregation when there are informational limitations.**

Supplement: Eligibility

$r = (r_d)_{d \in D} \in \{1,...,n\}^{\bar{d}}$ **Eligibility constraint**

Agent i is eligible for criterion $d \in D$ if

$\pi_d^{-1}(i) \le r_d$.

$C(r): X \to 2^N$ **Social choice rule with eligibility (SCRE)**

$|C(r)(I,q)| = \min[q, |\{i \in I \mid \text{agent } i \text{ is eligible}\}|]$.

SCRE fails to satisfy property (ii) in Axiom 3.

How can we extend a given SCR (with Axiom 3) to an SCRE?

Two perspectives: **Priority over eligible agents**

Compatibility between justification and eligibility

Priority over eligible agents:

Keep the original as much as possible.

Select the smallest number $\tilde{q} \geq q$ **satisfying**

$|C(I,\tilde{q}) \cup \{i \in I \mid \text{agent } i \text{ is eligible}\}|$

$= \min\,[q, |\{i \in I \mid \text{agent } i \text{ is eligible}\}|]$

We define $C(r)(I,q) = C(I,\tilde{q}) \cup \{i \in I \mid \text{agent } i \text{ is eligible}\}$.

A drawback is that an assigned agent i **is not eligible for the criterion** $\delta^{\gamma}(I,\tilde{q})(i)$ **that is used for justificastion.**

Compatibility between justification and eligibility:

Specify SCRE according to the following steps:

- **Set dummy agent ϕ.**
- **In step 1, the top-ranked agent among I at criterion $\gamma(1) \in D$ is selected. This agent is denoted by $i(1) \in I$.**
- **If agent $i(1)$ is eligible for $\gamma(1)$, they are assigned and justified by $\delta^{\gamma\dagger}(I, q)(i(1)) = \gamma(1)$.**
- **If they are not eligible for $\gamma(1)$, they are regarded as provisionally selected agents.**

- **At each step $k \geq 2$, the top-ranked agent at criterion $\gamma(k) \in D$ among remaining participants is selected. This agent is denoted by $i(k)$.**
- **Any provisionally selected agent i is assigned if they have better rank than agent $i(k)$ at criterion $\gamma(k)$ and are eligible for it. This agent i is justified by $\delta^{\gamma\dagger}(I,q)(i) = \gamma(k)$.**
- **If the selected agent $i(k)$ is eligible for $\gamma(k)$, they are assigned and justified by $\delta^{\gamma\dagger}(I,q)(i(k)) = \gamma(k)$.**

- **If they are not eligible for $\gamma(k)$, they are regarded as provisionally selected agents.**
- **We continue these steps until the number of assigned agents equals $\min[q, |\{i \in I \mid \text{agent } i \text{ is eligible}\}|]$.**
- **We specify SCRE $C^{\gamma\dagger}(r)(I,q)$ as the set of all assigned agents through these steps.**

Any assigned agent i is eligible for, and justified by, criterion $\delta^{\gamma\dagger}(I,q)(i)$.