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Impact of Financial Regulation and Innovation on Bubbles and Crashes due to Limited Arbitrage: Awareness Heterogeneity

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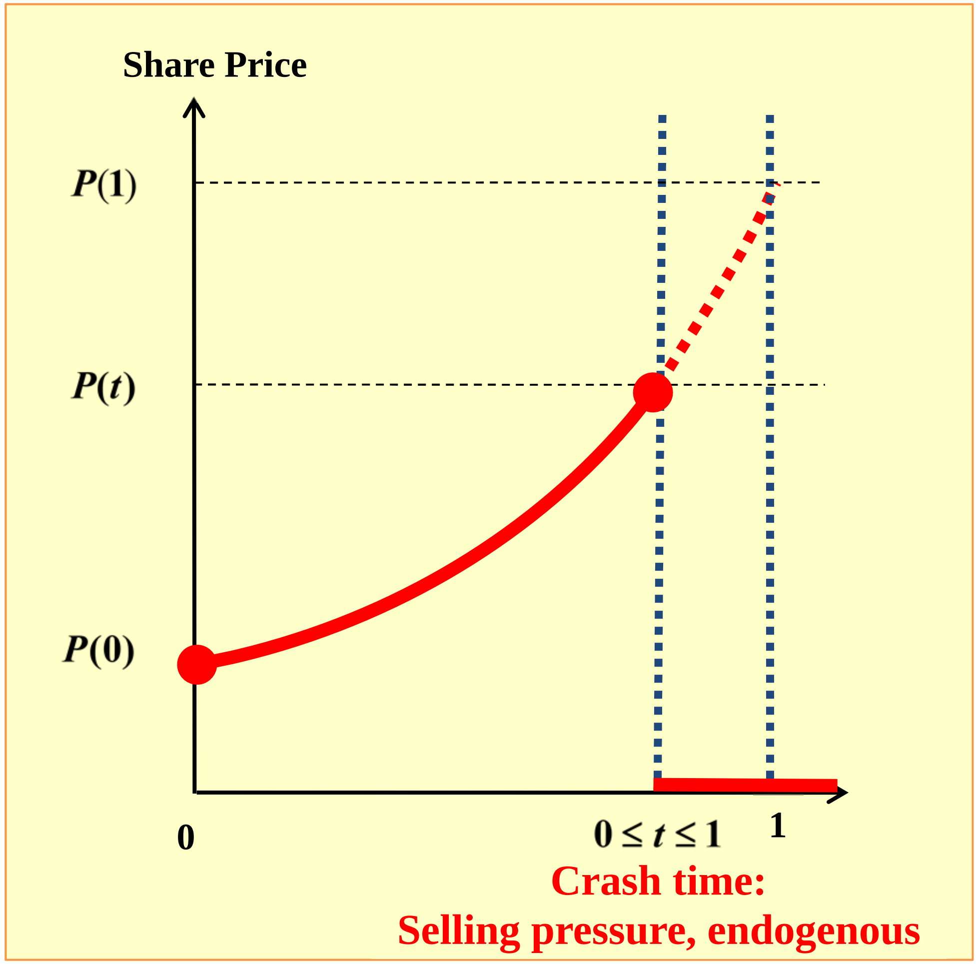
Bubbles and Crashes in a Company's Stock Market

Bounded time horizon $[0,1]$, Fundamental value zero



Limited Arbitrage Bubble

Shleifer and Vishny (92), Abreu and Brunnermeier (03), Matsushima (12), etc.

Rational traders (Arbitrageurs) interacts with
Irrational traders (Positive Feedback Traders, PFTs)

PFTs are slaves to Euphoria

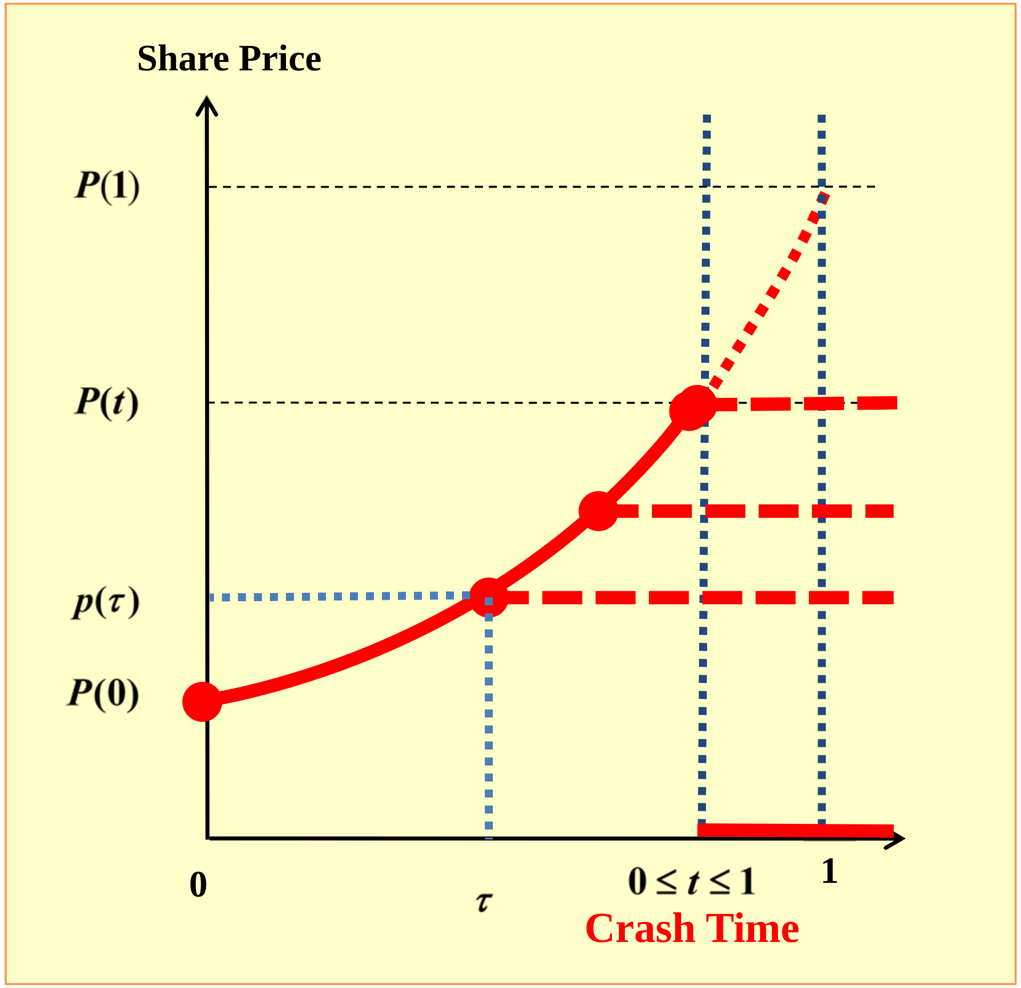
PFTs misperceive fundamental value greater than true value

PFTs reinforce misperception

Selling pressure makes mismatch between perception and share price
$\Rightarrow$ It dampens euphoria

PFTs are unaware of own reinforcement.

PFTs' Reinforcement Pattern



Arbitrageurs are almost certainly rational

Arbitrageur selects either ‘Time market’ or ‘Ride bubble’.
Arbitrageurs compete with one another: Earliest to time wins.

Relative future benefit $\dfrac{\textbf{Future benefit from 'Ride bubble'}}{\textbf{Instantaneous gain from 'Time market'}}$

is crucial for arbitrageurs’ incentives.

How to model their competition?

Timing Game with Behavioral Types: Matsushima (2012)

Arbitrageur is behavioral with small probability $\varepsilon > 0$.
Behavioral arbitrageur never times of her own accord.

Bubble is Harmful

Company: **Fund raising by share Issuance, leading to social harm (wasting funds)**
However, fund raising may make selling pressure and dampen euphoria

How can company issue shares without fear of crash?:

Encourage arbitrageurs to borrow money from PFTs!

Awareness Heterogeneity

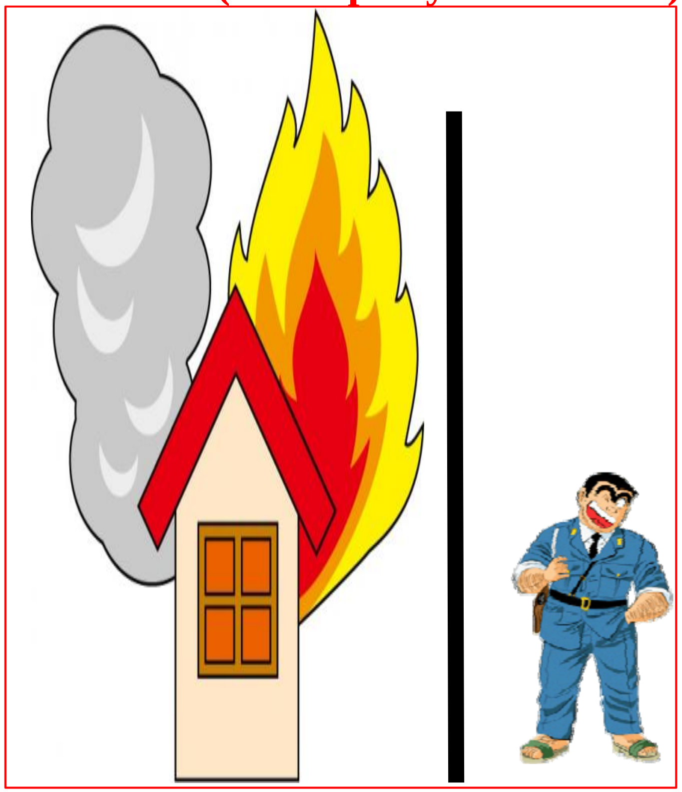
PFTs are unaware of euphoria, while arbitrageurs are aware of it.
$\Rightarrow$ **Arbitrageurs can borrow from PFTs with 'No Premium'.**

Question: How can we deter such harmful bubble?

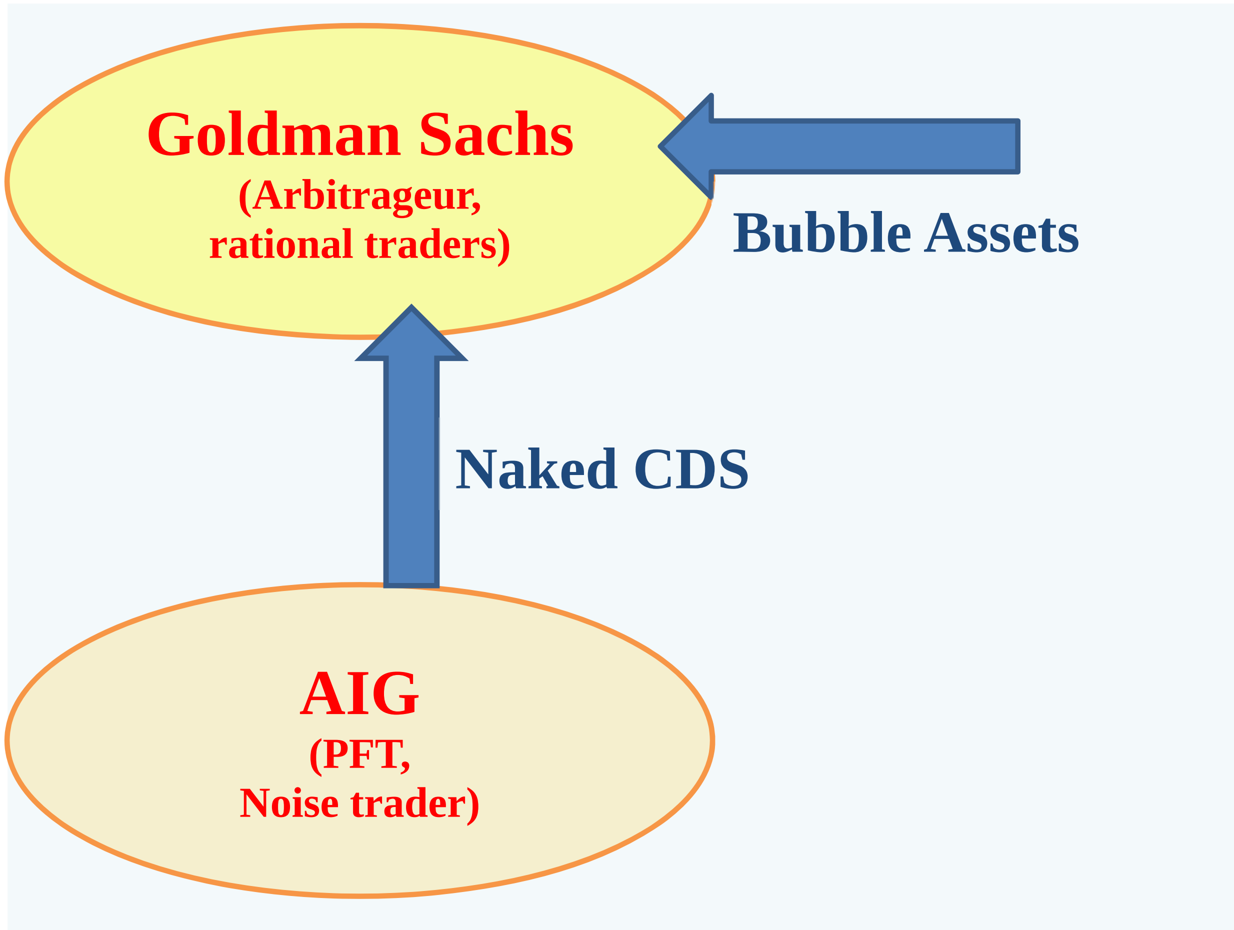
Two Methods: Financial regulation: Leverage ratio cap
Financial innovation: Credit Default Swap (CDS)

Covered CDS (Own default risk)

Naked CDS (Third party default risk)



Arbitrageur purchases naked CDS from PFT



Results (1)

Without CDS available, high leverage ratio fosters bubble.

With naked CDS available, high leverage ratio deters bubble.

With only covered CDS available, no-crash bubble is unique NE

Policy implication

With naked CDS, regulator should weaken leverage ratio cap even if company is unproductive.

Without CDS, regulator has dilemma caused by ignorance of productivity.

Results (2)

PFTs' capital growth is insufficient relatively to PFTs' loan growth:
(For example, high leverage ratio, enthusiastic PFTs)
Naked CDSs deters bubble.

PFTs' capital growth is sufficient relatively to PFTs' loan growth:
(For example, low leverage ratio, unenthusiastic PFTs)
Naked CDSs fosters bubble.

Policy Implication

Naked CDS is effective method in deterring social harm and even in fostering social benefit

Organization of This Paper

Formulation of Arbitrageur’s strategic competition

‘Timing Game with Behavioral Types’: Matsushima (2012)

Formulation of ‘Stock Market’

Incorporation of ‘Stock Market’ into ‘Timing Game with Behavioral Types’

Three Models: **Basic Model (No CDS)**
Covered CDS Model
Naked CDS Model

Timing Game with Behavioral Types

Players (Arbitrageurs) $i = 1, \ldots, n$
Each player selects time a_i **in time interval** $[0,1]$
Earliest to time wins

Assumptions: **Symmetric Game**
Winner payoff $\overline{v}_1(t)$ > **Loser payoff** $\underline{v}_1(t)$
$\overline{v}_1'(t) > 0$

Behavioral Types: **Player is behavioral with probability** $\varepsilon > 0$
Behavioral type never times, never wins.

Player is rational with remaining probability $1 - \varepsilon > 0$
Rational player randomizes time choice as best response

Strategy for rational arbitrageur

$q_i : [0,1] \to [0,1]$, $q_i(1) = 1$, non-decreasing.

Two Specifications

'Bubble and Crashes' strategy profile $\tilde{q} = (\tilde{q}_i)_{i \in N}$

'No Bubble' Strategy profile $q^* = (q_i^*)_{i \in N}$

'Bubbles and Crashes' Strategy Profile $\tilde{q}$

$\tilde{q}_i(\tilde{\tau}) = 0$: Player never times before critical time $\tilde{\tau} > 0$

$$\tilde{q}_1(t) = \frac{1 - \{1 - (1-\varepsilon)\tilde{q}_1(\tilde{\tau})\}\exp[-\frac{1}{n}\int_{\tau=\tilde{\tau}}^{t}\theta(\tau)d\tau]}{1-\varepsilon}$$ for all $t \in [\tilde{\tau}, 1]$:

Rational player times according to hazard rate $\theta(t)$ after $\tilde{\tau}$.

Hazard rate: $$\theta(t) \equiv \frac{n}{n-1}\frac{\overline{v}_1'(t)}{\overline{v}_1(t) - \underline{v}_1(t)}$$

Critical time $\tilde{\tau}$: $$\varepsilon = \exp[-\frac{1}{n}\int_{\tau=\tilde{\tau}}^{1}\theta(\tau)d\tau]$$

Theorem 1:

'Bubble and Crash' strategy profile $\tilde{q}$ is NE if and only if

$$I_1 \equiv \exp[-\frac{1}{n}\int_{\tau=0}^{1}\theta(\tau)d\tau] \leq \varepsilon$$

$\tilde{q}$ is unique NE if

$$I_1 < \varepsilon$$

Index I_1 implies a degree of (inverse of) relative future benefit

Smaller I_1, more likely bubble

'Bubbles and Crashes' as Unique NE

‘No Bubble’ Strategy Profile q^*

$q_i^*(0)=1$: **Rational player certainly times market at initial time 0.**

Theorem 2:

‘No Bubble’ Strategy Profile q^* is NE if and only if

$$I_2 \equiv \frac{\overline{v}_1(0)-\underline{v}_1(0)}{\overline{v}_1(1)-\overline{v}_1(0)} \geq \Big[\sum_{1\leq l\leq n-1} \frac{(n-1)!}{l!(n-1-l)!}\Big(\frac{1-\varepsilon}{\varepsilon}\Big)^l \frac{1}{l+1}\Big]^{-1}.$$

Index I_2 implies another degree of (inverse of) relative future benefit

Smaller I_1, more likely bubble

What do indices I_1 and I_2 imply?

We need to specify winner and loser payoffs by formulating stock market

Formulation of Stock Market

Assumptions: Market interest rate zero, No dividend, Short-sale prohibited

Company: Total share $S(t)$: Share issuance $S'(t)\Delta > 0$ from t to $t+\Delta$

PFTs: Sufficient Personal Capital $B(t)$
PFTs misperceive $P(t)$ unchanged over time.
PFTs unconsciously reinforce misperception: $P'(t) > 0$

Bubble crashes once arbitrageurs' share become less than $n\phi \times 100\,\%$.

Arbitrageurs: Homogeneous: $S_i(t) = S_1(t)$
Share purchase $S_i'(t)\Delta > 0$ from t to $t+\Delta$

Company issue shares as much as possible: $S_i(t) = \phi S(t)$

Awareness Heterogeneity:

Arbitrageurs can make short-term non-recourse debt contract (collateralized by shares) with PFTs with no premium

Leverage ratio cap $L > 1$:

Arbitrageurs will borrow at maximum:

Arbitrageur's debt obligation:

$$\frac{L-1}{L} P(t) S_i(t)$$

Arbitrageur's Personal Capital:

$$W_i(t) = P(t) S_i(t) - \frac{L-1}{L} P(t) S_i(t) = \frac{P(t) S_i(t)}{L}$$

$$\therefore \quad W_i'(t) = \frac{P(t) S_i'(t) + P'(t) S_i(t)}{L} \quad \text{.........} \ \text{(A)}$$

Arbitrageur earns capital gain $S_i(t)\{P(t+\Delta)-P(t)\}$ **from** t **to** $t+\Delta$**:**

$$\therefore \quad W_i(t+\Delta)-W_i(t)=S_i(t)\{P(t+\Delta)-P(t)\}$$

$$\therefore \quad W_i'(t)=S_i(t)P'(t) \quad \text{......... (B)}$$

From (A) and (B), we can derive:

$$\frac{P(t)S_i'(t)+P'(t)S_i(t)}{L}=S_i(t)P'(t)$$

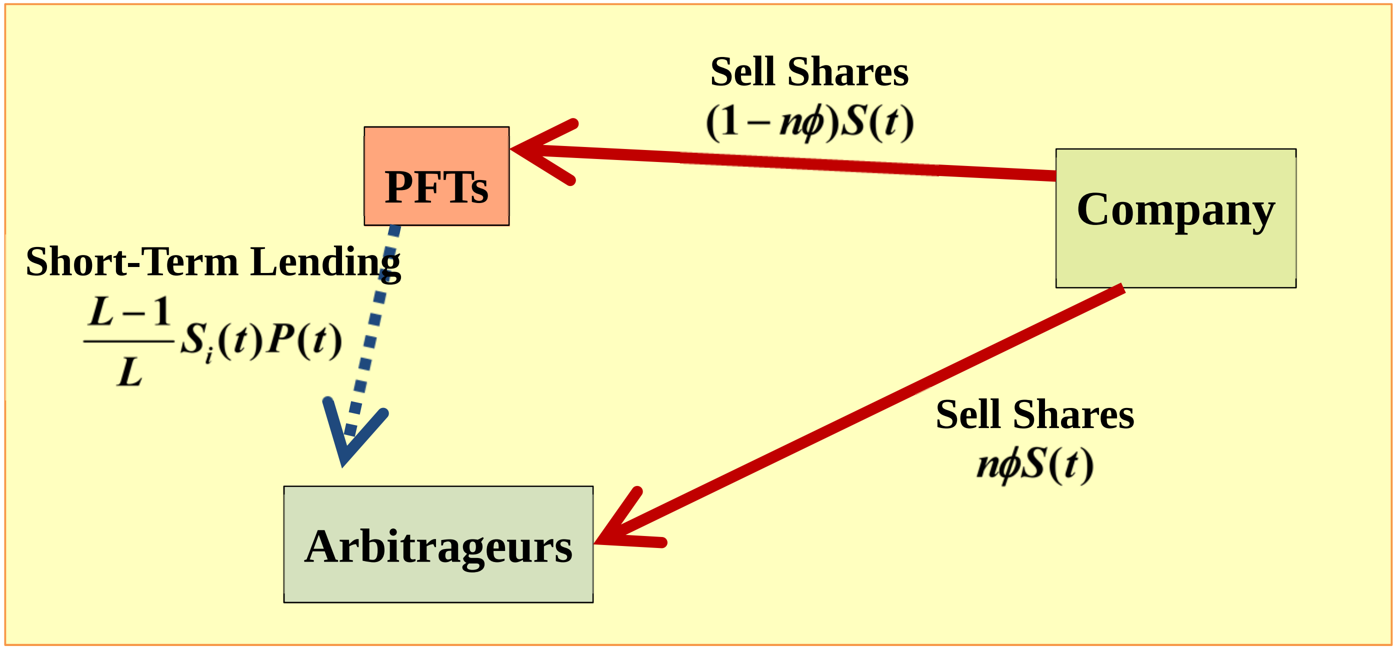
Total Share: $S(t)=S(0)\left(\frac{P(t)}{P(0)}\right)^{L-1}$

Arbitrageur's Share: $S_i(t)=\phi S(0)\left(\frac{P(t)}{P(0)}\right)^{L-1}$

Arbitrageur's Personal Capital: $W_i(t)=\frac{P(t)S_i(t)}{L}=\frac{\phi}{L}P(0)S(0)\left(\frac{P(t)}{P(0)}\right)^{L}$

Incorporation of ‘Stock Market’ into ‘Timing Game with Behavioral Type’

1) Basic Model: No CDS Available



Winner payoff: $\bar{v}_i(t) = W_i(t)$

Loser Payoff: $\underline{v}_i(t) = 0$

$$\theta^*(t,L) = L\frac{n}{n-1}\frac{P'(t)}{P(t)} \qquad \text{increasing in } L \text{ and } \frac{P'(t)}{P(t)}$$

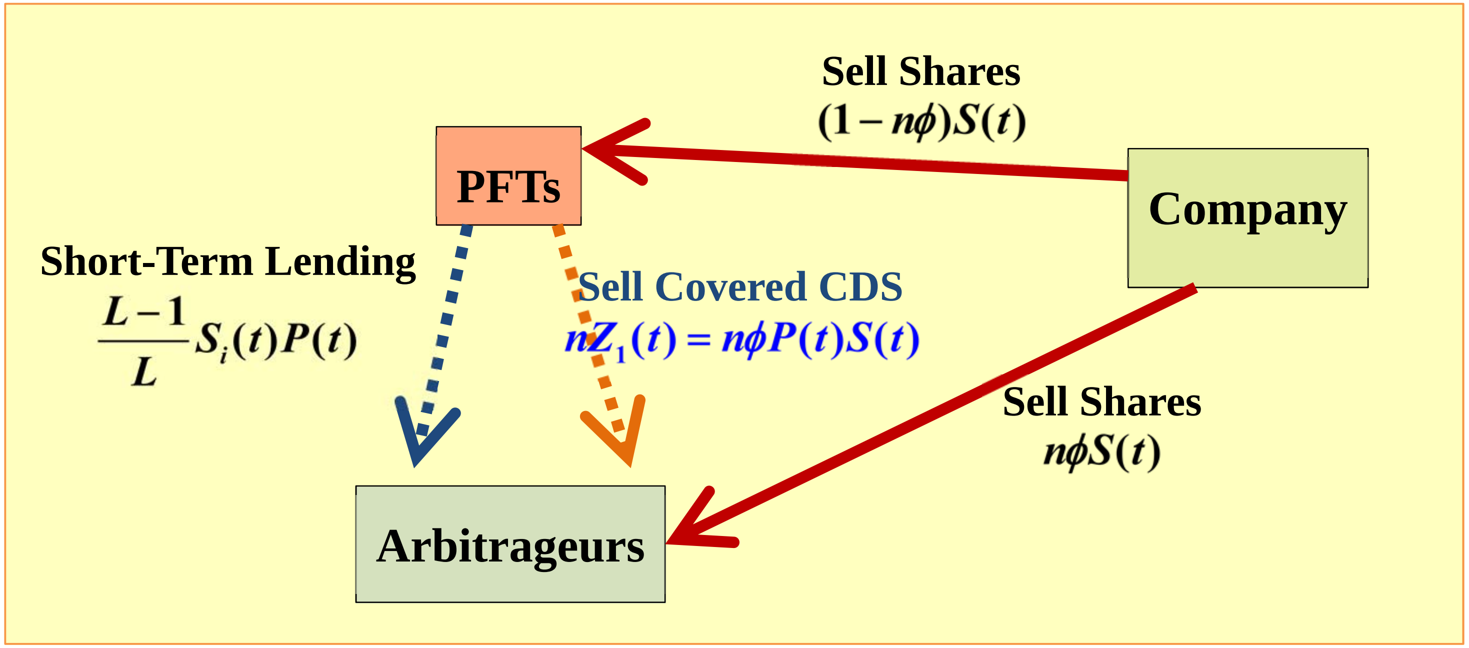
$$I_1^*(L) = \left(\frac{P(0)}{P(1)}\right)^{\frac{L}{n-1}} \qquad \text{decreasing in } L \text{ and } \frac{P(1)}{P(0)}$$

$$I_2^*(L) = \frac{1}{\left(\frac{P(1)}{P(0)}\right)^{L} - 1} \qquad \text{decreasing in } L \text{ and } \frac{P(1)}{P(0)}$$

More enthusiastic, more likely bubble.
Greater leverage ratio L, more likely bubble.

Even with tiny enthusiasm, high leverage ratio fosters bubble!

2) Covered CDS Model



Covered CDS is insurance against own default risk

Payment of Covered CDS: $Z_i(t) = P(t)S_i(T) = \phi P(t)S(t) = LW_i(t)$

Winner never receives $Z_i(t)$

Loser receives $Z_i(t) = LW_i(t)$ **and pays debt obligation** $(L-1)W_i(t)$.

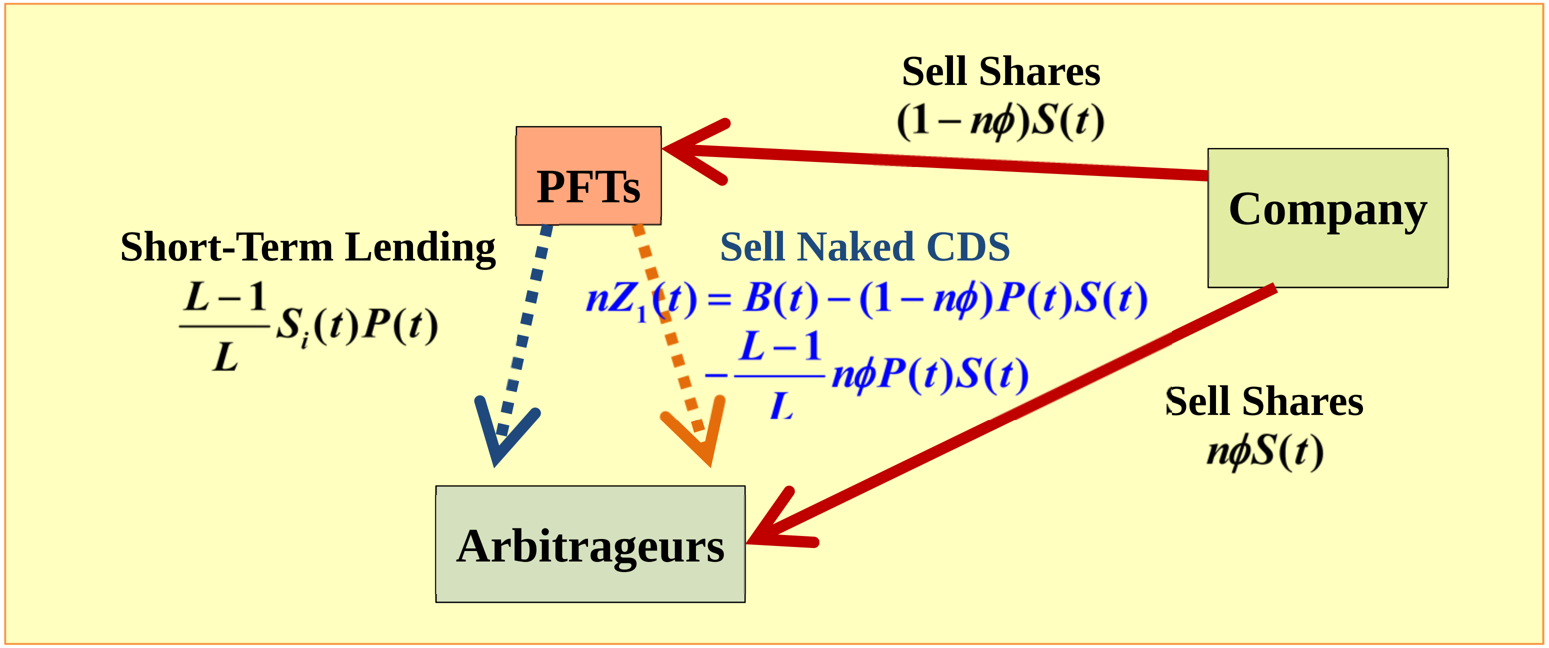
Winner payoff: $\overline{v}_i(t) = W_i(t)$

Loser Payoff $\underline{v}_i(t) = Z_i(t) - (L-1)W_i(t) = W_i(t)$

Winner and Loser payoffs are equivalent: $\overline{v}_i(t) = \underline{v}_i(t)$

$\Rightarrow$ **No-Crash Bubble is Unique NE**

3) Naked CDS Model



Naked CDS is speculative instrument against third party default risk

Both winner and losers can receive payment $Z_i(t)$

Arbitrageur can demand naked CDS without underlying Shares.

Arbitrageurs strategically save demand in order to prevent increase in naked CDS price (positive premium) from dampening euphoria.

Hence, payment from naked CDS is given by:

Payment of naked CDS = PFTs' Personal Capital - Shareholding Value - Loan

$$nZ_1(t) = B(t) - (1 - n\phi)P(t)S(t) - \frac{L-1}{L} n\phi P(t)S(t)$$

Rapid growth of PFTs' capital $B(t)$ $\Leftrightarrow$ **Rapid growth of naked CDS** $nZ_1(t)$

Winner Payoff:

$$\overline{v}_i(t) = W_i(t) + Z_i(t) = \frac{1}{n}B(t) - (\frac{1}{n} - \frac{2\phi}{L})P(0)S(0)(\frac{P(t)}{P(0)})^L$$

Loser Payoff:

$$\underline{v}_i(t) = Z_i(t) - (L-1)W_i(t) = \frac{1}{n}B(t) - \{\frac{1}{n} + \frac{(L-2)\phi}{L}\}P(0)S(0)(\frac{P(t)}{P(0)})^L$$

Difference between Winner and Loser Payoffs:

$$\overline{v}_i(t) - \underline{v}_i(t) = LW_i(t)$$

Relative future benefit depends on
growth balance between capital $B(t)$ and Loan $(L-1)W_i(t)$

Theorem 3 (1):

$$\text{If } Z_1'(t) < (L-1)W_1'(t) \text{ for all } t \in [0,1], \text{ then}$$

$$\theta^{**}(t,L) > \theta^{*}(t,L),\ \ I_1^{**}(L) > I_1^{*}(L),\ \ I_2^{**}(L) > I_2^{*}(L).$$

If payment of naked CDS grows less rapidly than loan to arbitrageurs, bubble is less likely in naked CDS model than basic model.

If leverage ratio is sufficient and PFTs are enthusiastic, naked CDS deters bubble.

Theorem 3 (2):

$$\text{If } Z_1'(t) > (L-1)W_1'(t) \text{ for all } t \in [0,1], \text{ then}$$

$$\theta^{**}(t,L) < \theta^{*}(t,L),\ \ I_1^{**}(L) < I_1^{*}(L),\ \ I_2^{**}(L) < I_2^{*}(L).$$

If payment of naked CDS grows more rapidly than loan to arbitrageurs, bubble is more likely in naked CDS model than basic model.

If leverage ratio is insufficient and PFTs are not very enthusiastic, naked CDS fosters bubble.

Policy Implication of Theorem 3

Naked CDSs deters bubble if there is major concern about social harm.

Naked CDS fosters the bubble if there is little concern about it.
Bubble is beneficial as supplementary for financial friction.

Naked CDS is effective policy method in
deterring social harm, and even in fostering social benefit.

Theorem 4:

Suppose $L > 2n\phi$. Then

$$\frac{\partial}{\partial L}\theta^{**}(t,L) < 0,\quad \frac{\partial}{\partial L}I_1^{**}(L) > 0, \text{ and } \quad \frac{\partial}{\partial L}I_2^{**}(L) > 0.$$

High leverage deters bubble in Naked CDS Model.

$\because$ **High leverage ratio crows out future reserve for naked CDS.**

Policy Implication of Theorem 4

With naked CDS, regulator can set high leverage ratio irrespective of productively.
Without naked CDS, regulator has dilemma caused by ignorance of productivity

Summary of Results

	Availability Impact			High Leverage Ratio (Weak Cap)		
	Emergence	Persistence	Intrinsic Social Cost	Emergence	Persistence	Intrinsic Social Cost
No CDS				+	+	+
Covered CDS	++	++ (No Crash)	+	0	0	+
Naked CDS	Capital Growth >relative Loan Growth Low leverage ratio Weak enthusiasm + Capital Growth < Loan Growth High leverage ratio Strong enthusiasm —	Capital Growth >relative Loan Growth Low leverage ratio Weak enthusiasm + Capital Growth < Loan Growth High leverage ratio Strong enthusiasm —	Capital Growth > Loan Growth (Low leverage ratio Weak enthusiasm) + Capital Growth < Loan Growth High leverage ratio Strong enthusiasm —	—	—	?

Theories of Bubbles (Immature, Complementary)

Fiat Money Bubble:	**Tirole (85)**
Lemon Bubble:	**Allen and Gorton (93)**
Higher Order Belief Bubble:	**Morris and Shin (01), Abreu and Brunnermeier (03)**
Heterogeneous Belief Bubble:	**Harrison and Kreps (78)**
Limited Arbitrage Bubble:	**Shleifer and Vishny (92), Abreu and Brunnermeier (03), Matsushima (12)**

Related Literatures

Limited Arbitrage:	**Previous works never examined harmful bubbles.**
Prior Heterogeneity:	**Optimists are CDS sellers and borrowers, while PFTs are sellers but lenders.**
	Arbitrageurs have option to solve awareness heterogeneity
Fostel et al (12):	**GE with prior heterogeneity**
	‘Unexpected’ introduction of naked CDS increases default risk.
Hart et al (11):	**CDS price as informative signal about default risks**
Empirical facts:	**US housing bubble and impact of naked CDS in 05 and 06**
	Stein (96) etc.: Impact of mispricing on real investment
	Baker et al (10): Company with limited debt capacity is sensitive to stock price.

Fostel et al (2012): Heterogeneous belief bubble

Impact of unexpected introduction of naked CDS on Sudden Death

Optimists purchase bubble asset, and sell naked CDS to pessimists.

Our paper: Limited arbitrage bubble

Impact of established naked CDS market on deterrence of bubble

Arbitrageurs purchase bubble asset, and purchase naked CDS from PFTs.

Related Episode:

Goldman Sachs purchase bubble asset, and purchase naked CDS from AIG.

This episode: Naked CDS as unsecured credit, unexpected introduction
Our paper: Naked CDS as secured credit, already in place